

Inelastic and deep inelastic neutron spectroscopy of water under ultra-confinement*

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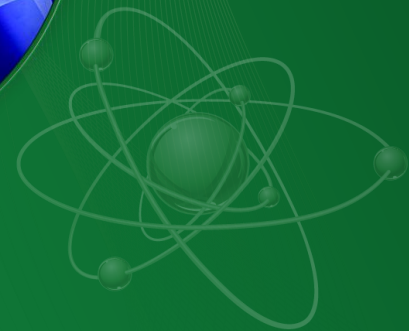
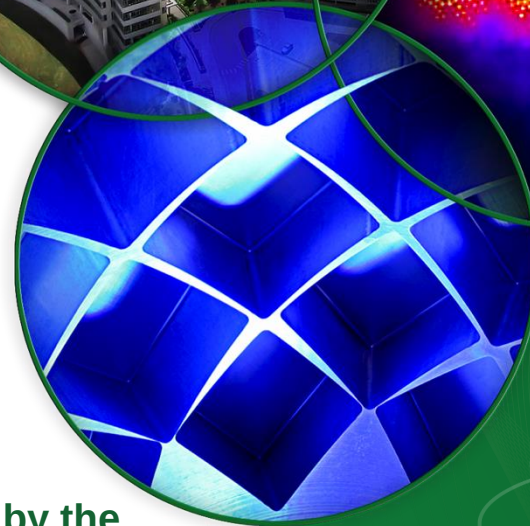
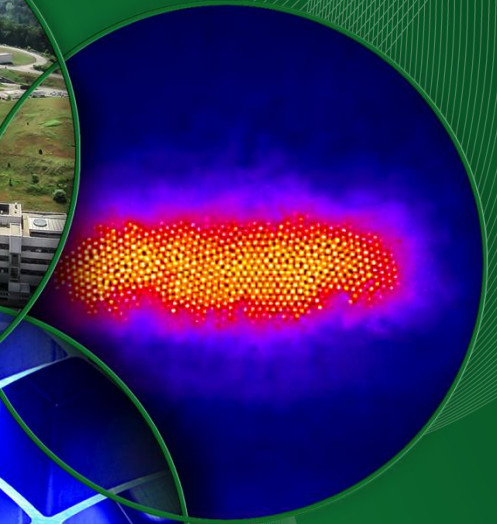
T. Prisk – NIST, MD, USA

A.G. Seel – University of Oxford, UK

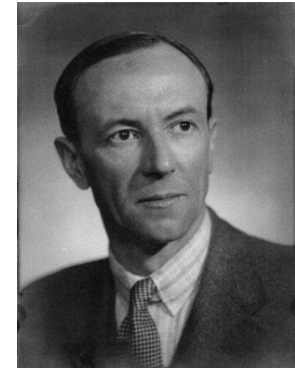
M. Krzystyniak, G. Romanelli – ISIS, RAL, UK

*The neutron experiment at ORNL was sponsored by the Sci. User Facilities Div., BES, U.S. DOE. This research was sponsored by the Div. Chemical Sci., Geosciences, and Biosciences, BES, U.S. DOE. The STFC RAL is thanked for access to ISIS neutron facilities.

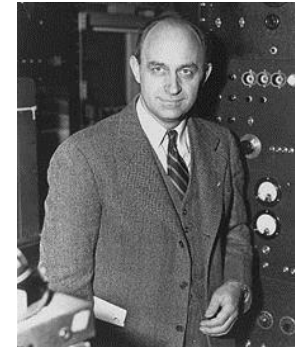
ORNL is managed by UT-Battelle
for the US Department of Energy



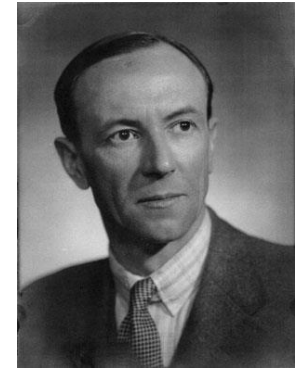
The 85th anniversary of the discovery of the neutron by James Chadwick



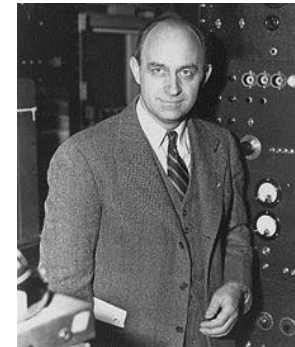
The 80th anniversary of the patent issued by the Ministero delle Corporazioni to Enrico Fermi for the use of neutrons to enhance artificial radioactivity



The 85th anniversary of the discovery of the neutron by James Chadwick



The 80th anniversary of the patent issued by the Ministero delle Corporazioni to Enrico Fermi for the use of neutrons to enhance artificial radioactivity



November 7th this year marks the 150th birthday of Marie Curie, who is one of the most famous physical chemists of all time, and was awarded two Nobel Prizes for her work in radioactivity



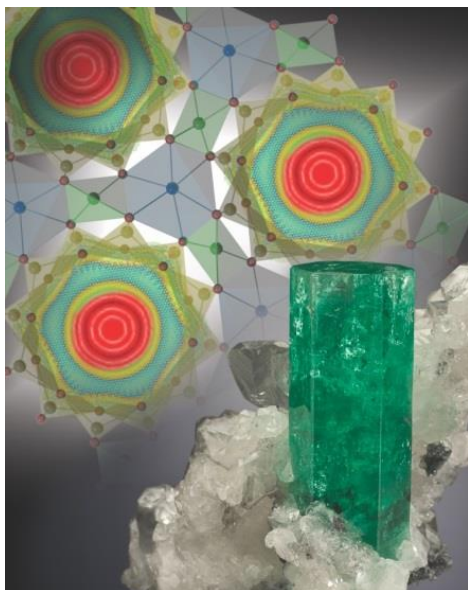
SNS, ORNL



Graphite Reactor

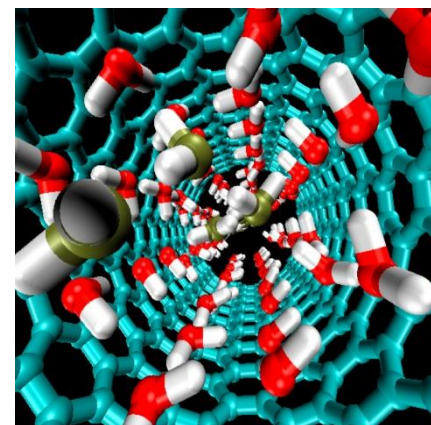
HFIR, ORNL



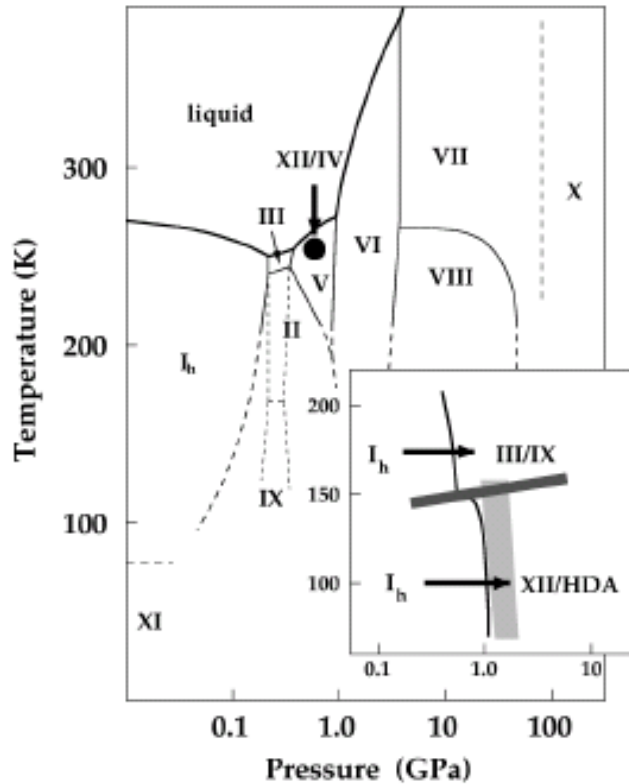


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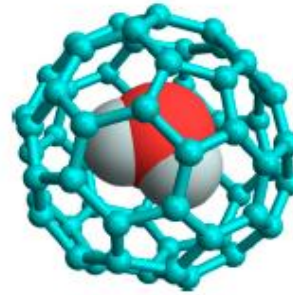
- **Introduction**
- **INS study of water in beryl,
Water tunneling**
- **DINS study of beryl**
- **Dynamics of water in
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- **Summary**



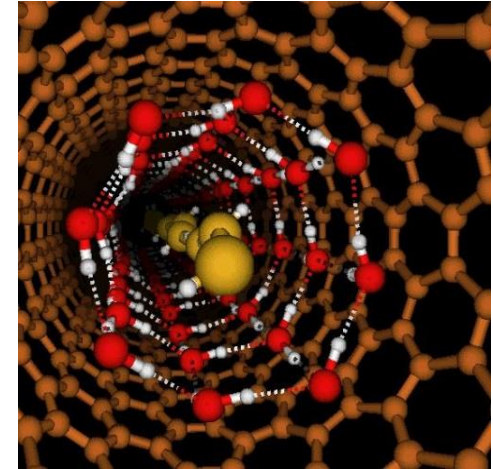
Examples of different bulk and confined water



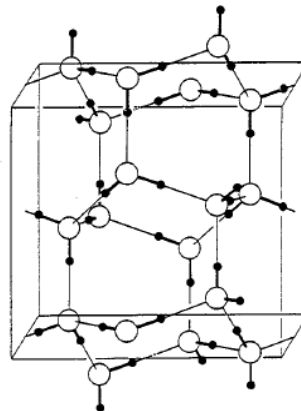
The phase diagram of ice



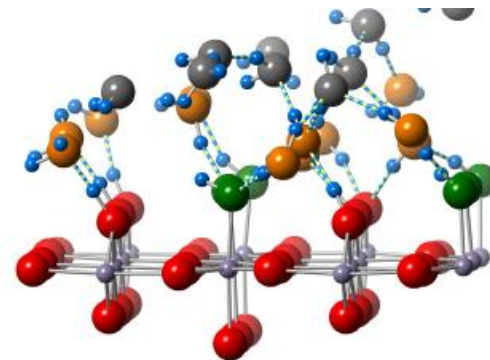
H₂O water in fullerene C₆₀



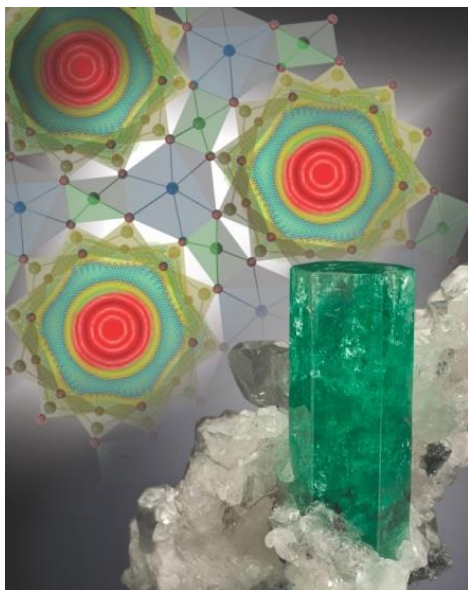
Water in SWNT



Ice-Ih

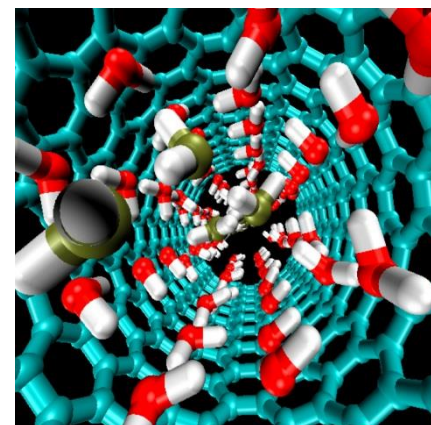


Water on surface of cassiterite (SnO₂)

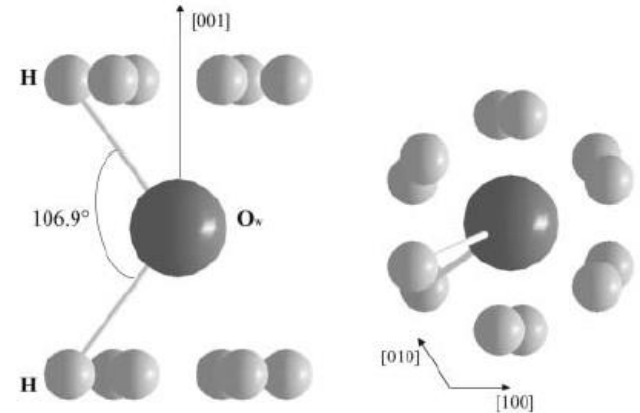
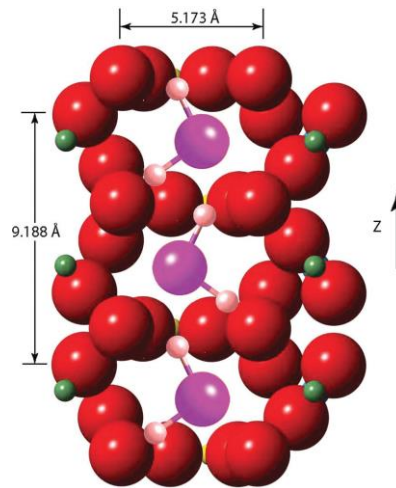
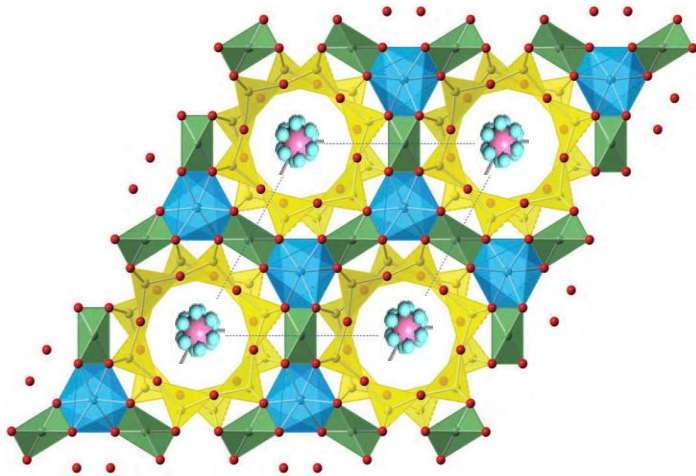


OUTLINE

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Beryl



(Left) Structure of beryl ($\text{Be}_3\text{Al}_2\text{Si}_6\text{O}_{18} \cdot 0.6\text{H}_2\text{O}$, space group *P6/mcc*) showing channels running parallel to the crystallographic *c*-axis (out of the figure's plane): Si, yellow; Al, blue; O, red; and Be, green.

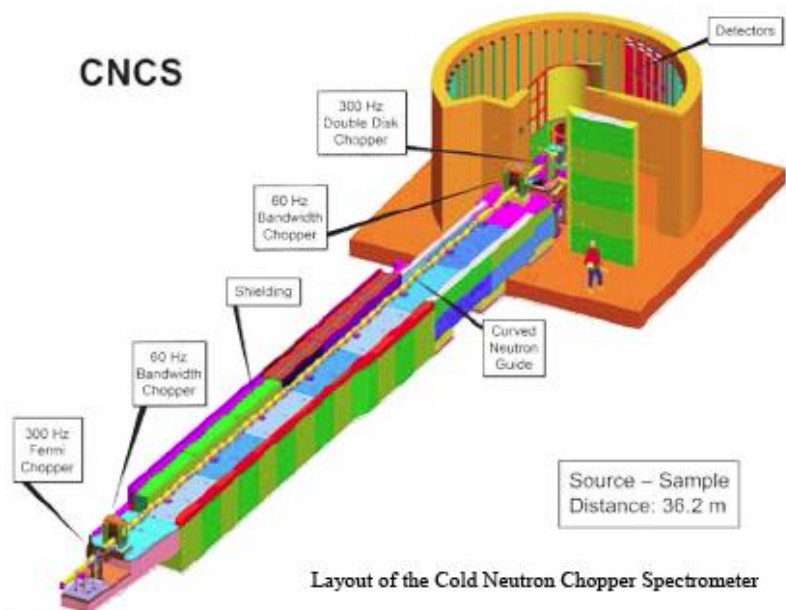
(Middle) Graphical representation of the alignment of water molecules inside the channels, beryl oxygen, red; water oxygen, dark pink; water hydrogen, light pink; beryllium, green; and silicon, yellow.

Topological configuration of water molecules into the channel of the alkali-poor beryl, viewed down [100] (left) and down [001] (right).

G.D. Gatta *et al.* Am. Mineral. 91 (2006) 29

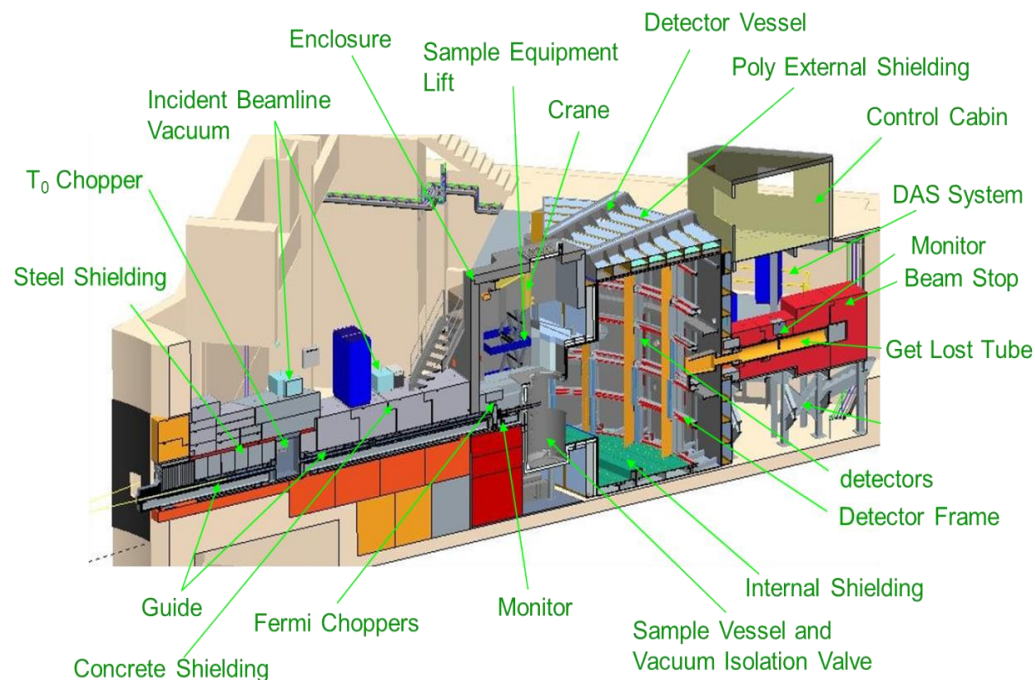
CNCS: Cold Neutron Chopper Spectrometer

CNCS

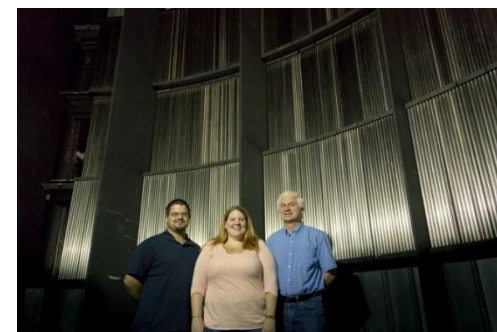


$E_i = <40 \text{ meV}$
 Scattering angles:
 $-50 \text{ to } 140^\circ \text{ (h)}, \pm 16^\circ \text{ (v)}$

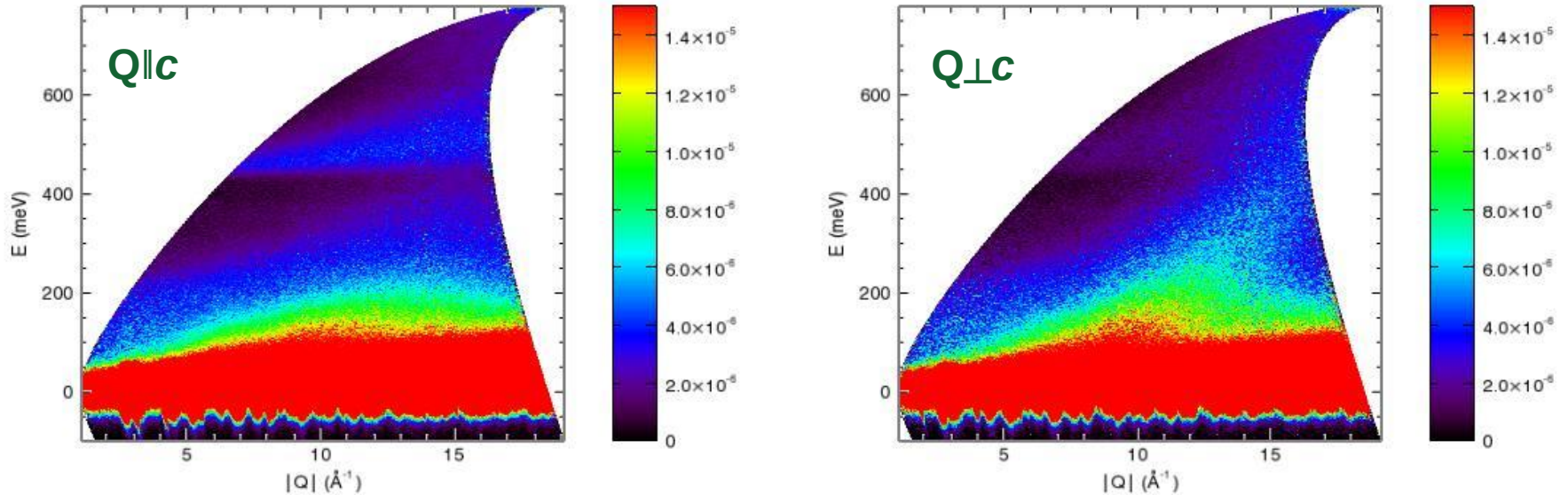
SEQUOIA: The Fine Resolution Fermi Chopper Spectrometer



$E_i = 4 - 6000 \text{ meV}$
 Scattering angles:
 $-30 \text{ to } 60^\circ \text{ (h)}, \pm 18^\circ \text{ (v)}$



S(Q,E) spectra of beryl, SEQUOIA



(Q,E) contour plots of S(Q,E) spectra for the neutron momentum transfer Q being perpendicular and parallel to crystallographic c-axis. The INS data were collected at T=6 K with $E_i=800$ meV.

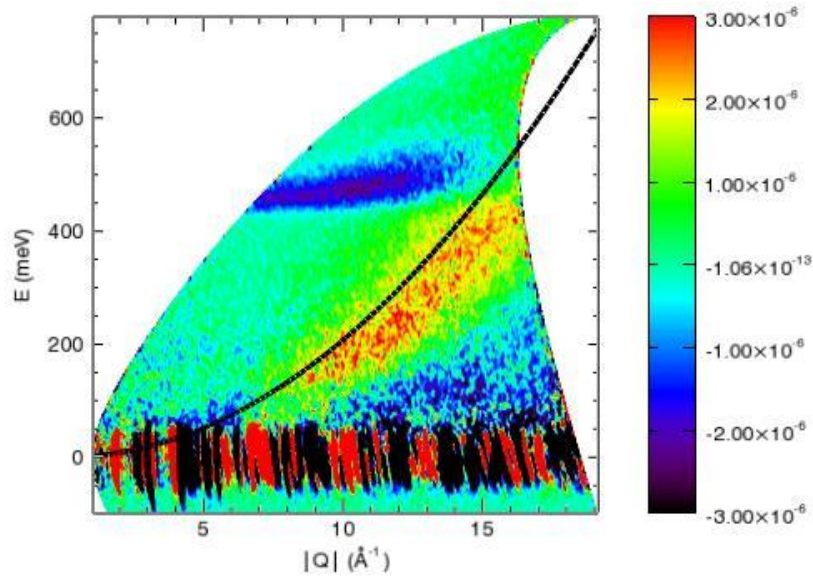
$$S_{1ph}^{inc}(\mathbf{Q}, E) = \sum_i \frac{\sigma_i \hbar^2}{6NEM_i} \exp[-(\mathbf{Q}\mathbf{u}_i)^2] [n_B(E, T) + 1] \sum_{j, \mathbf{q}} |\mathbf{Q}\mathbf{e}_i(j, \mathbf{q})|^2,$$

$$(\mathbf{Q}\mathbf{u}_i)^2 = \int \sum_{j, \mathbf{q}} \frac{\hbar^2}{6NEM_i} |\mathbf{Q}\mathbf{e}_i(j, \mathbf{q})|^2 [2n_B(E, T) + 1] \delta[E - E(j, \mathbf{q})] dE$$

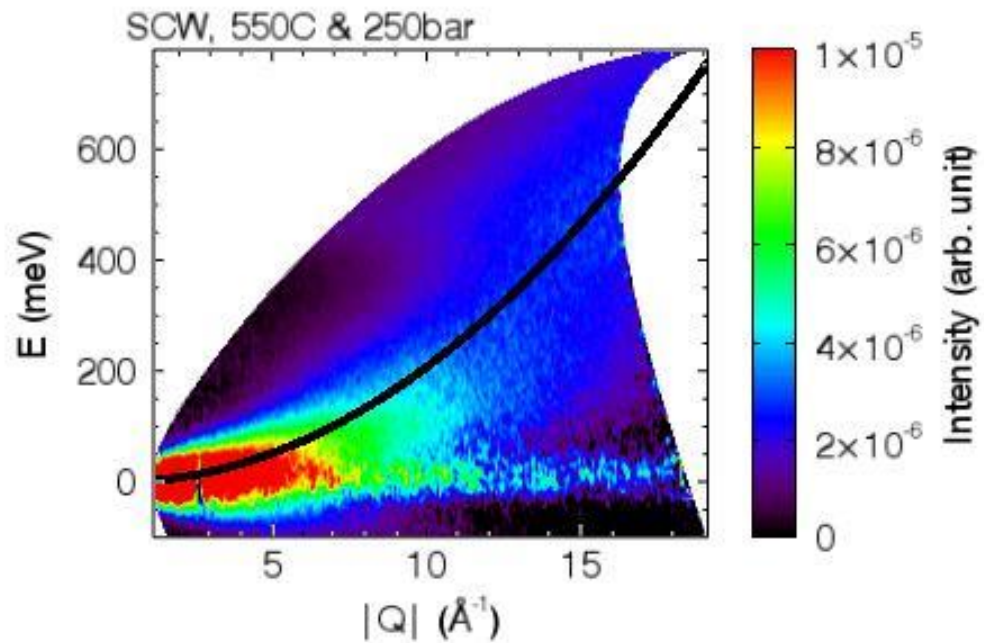
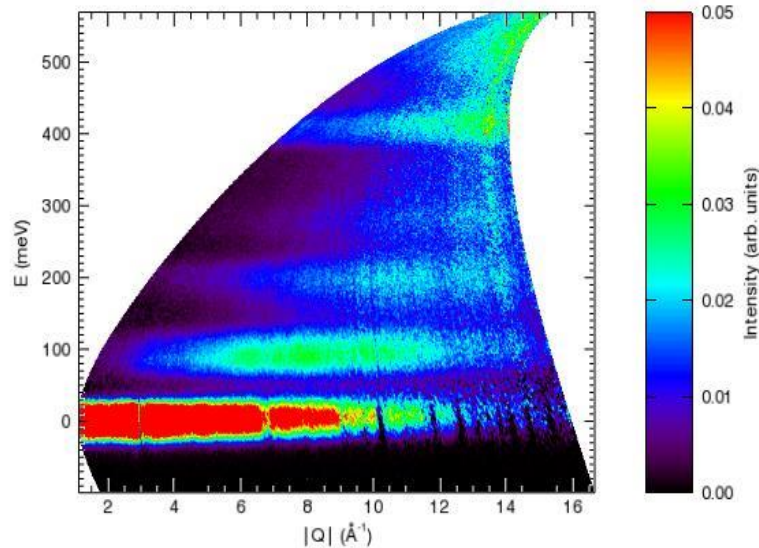
L.M. Anovitz *et al.*, Phys. Rev. E 88 (2013) 052306

A.I. Kolesnikov *et al.*, J. Phys. Chem. B 118 (2014) 13414

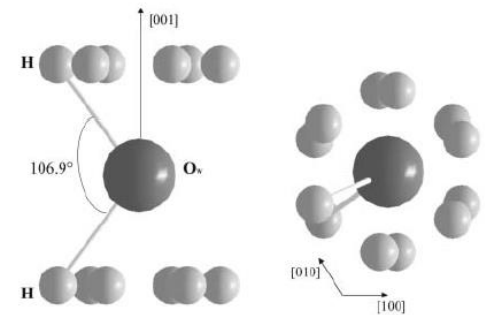
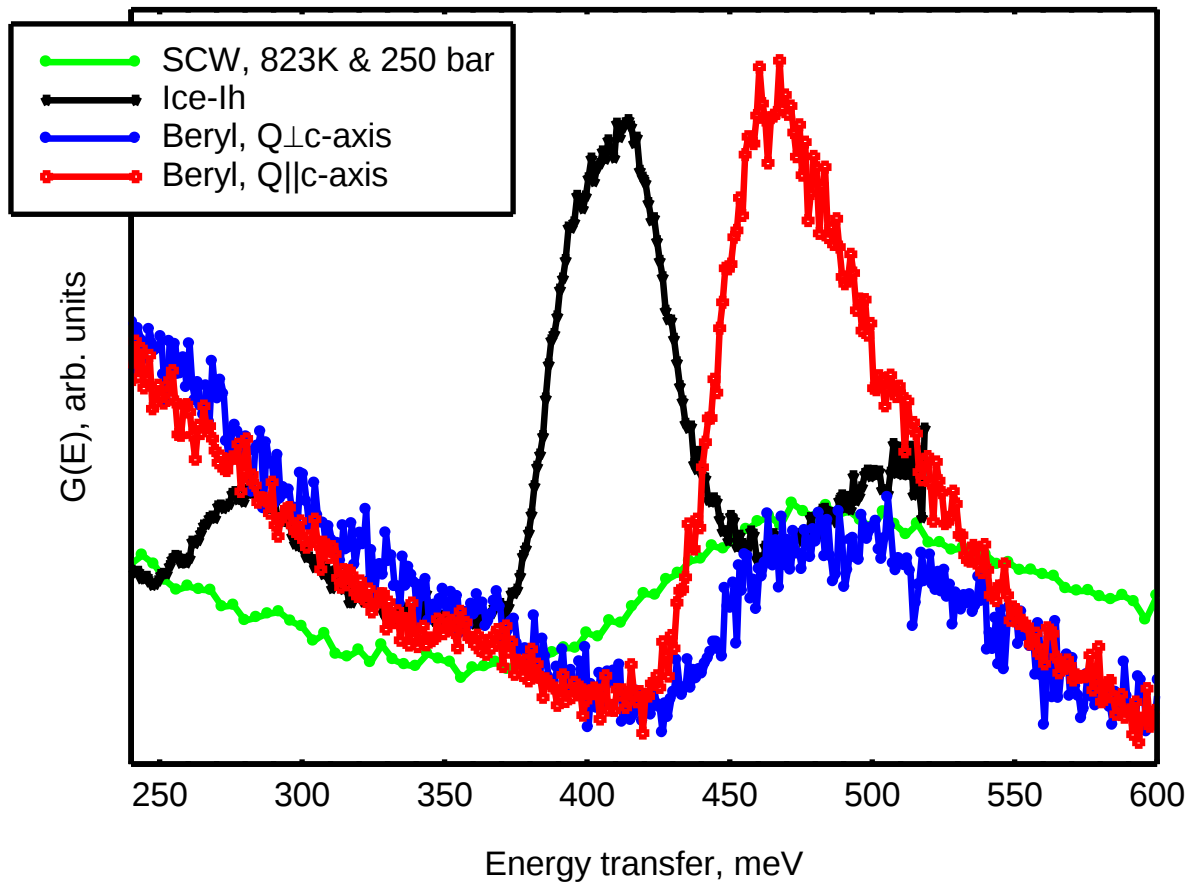
Beryl



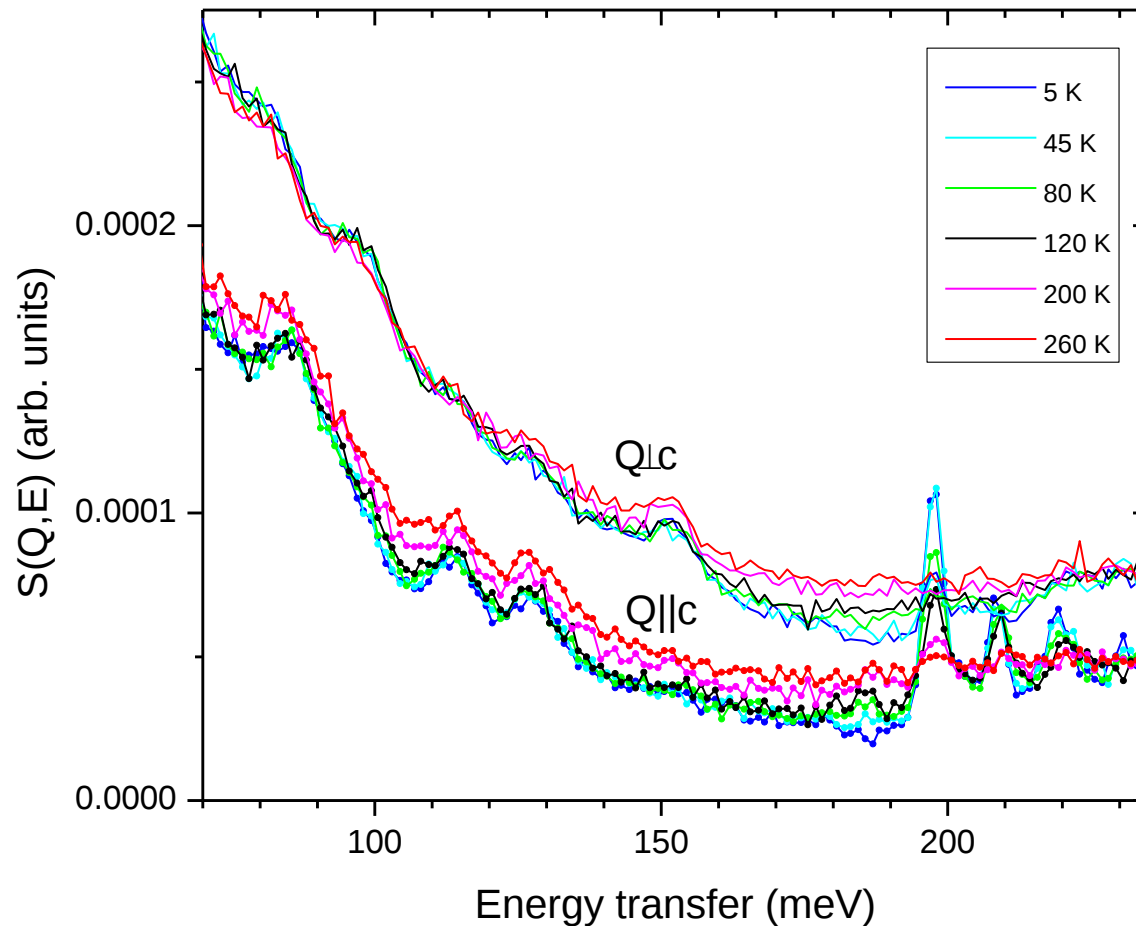
Difference between the $S(Q,E)$ spectra from beryl at 6 K for $Q \perp c$ and $Q \parallel c$ orientations. The solid line shows the neutron recoil spectrum for free particle of mass $m=1$ a.u., $E=\hbar^2 Q^2/2m$.



INS spectra of ice-Ih at $T=6$ K (left) and supercritical water at $T=280$ K and $P=250$ bar (top, right).

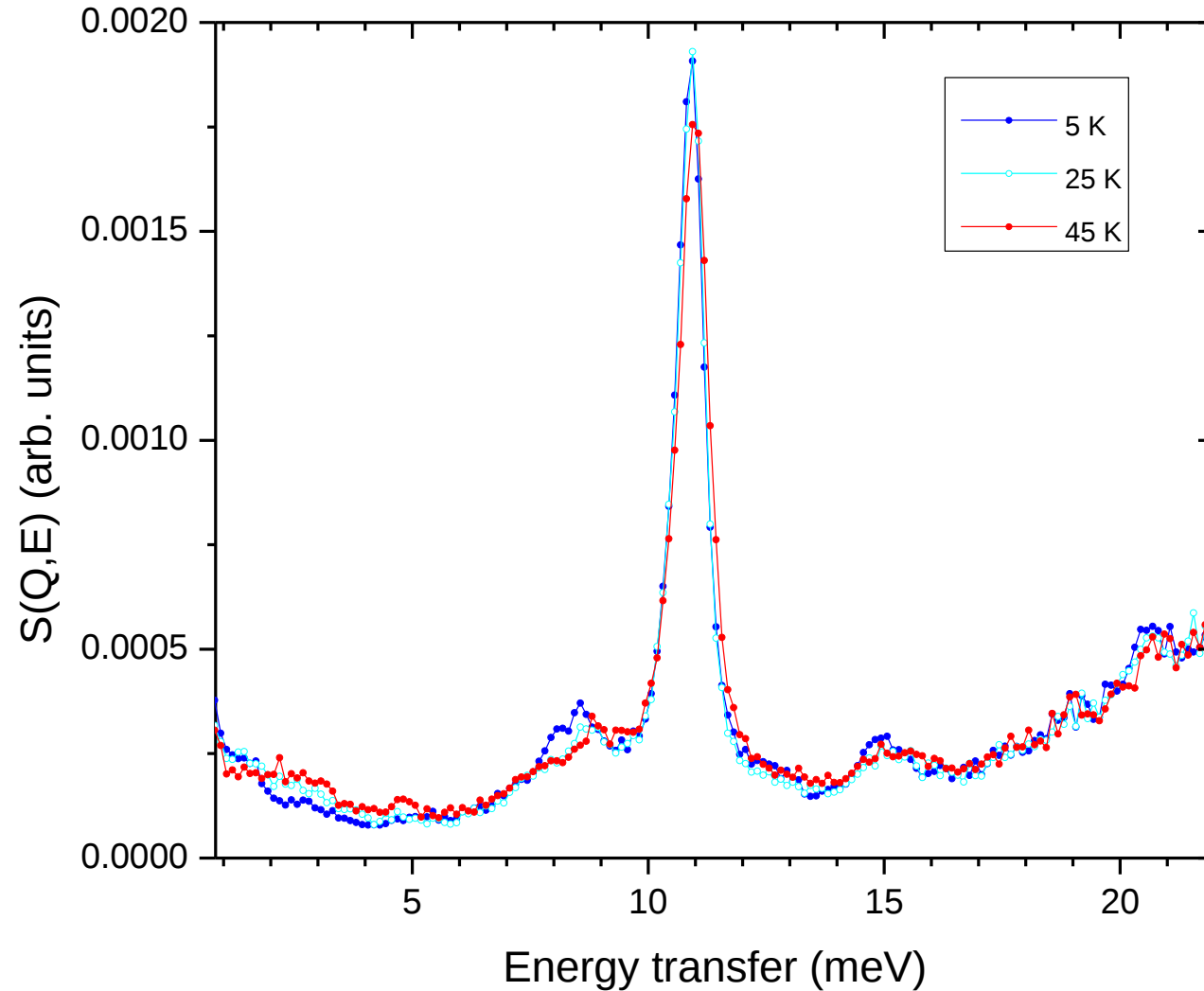


INS spectra at $T=6$ K for beryl measured with neutron momentum transfer Q parallel and perpendicular to the crystallographic c -axis; $E_i=800$ meV, compared to supercritical water at $T=823$ K & $P=250$ bar, and ice-Ih at $T=6$ K ($E_i=600$ meV). The confined water intramolecular O-H stretching peak is at 465 meV.

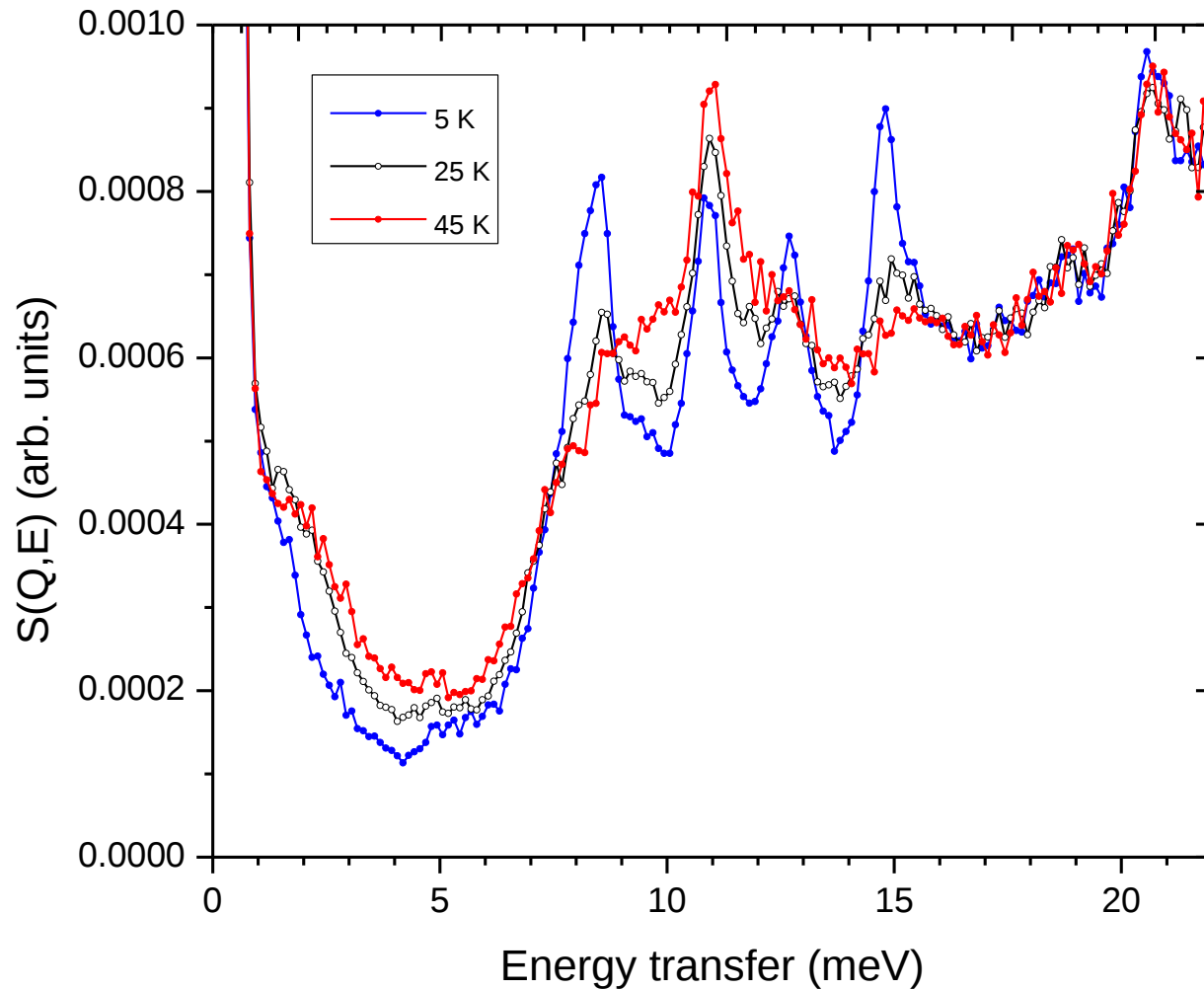


INS spectra for $E_i=250$ meV. The peak at 197 meV is due to H-O-H bending mode (ν_2), $E_{\text{bend}}=197.3+0.26(459.7-E_{\text{str}})$, [1]. Three peaks at the right are due to multi-phonon neutron scattering involving ν_2 and translational modes (~ 11 meV) of water molecule along the c-axis.

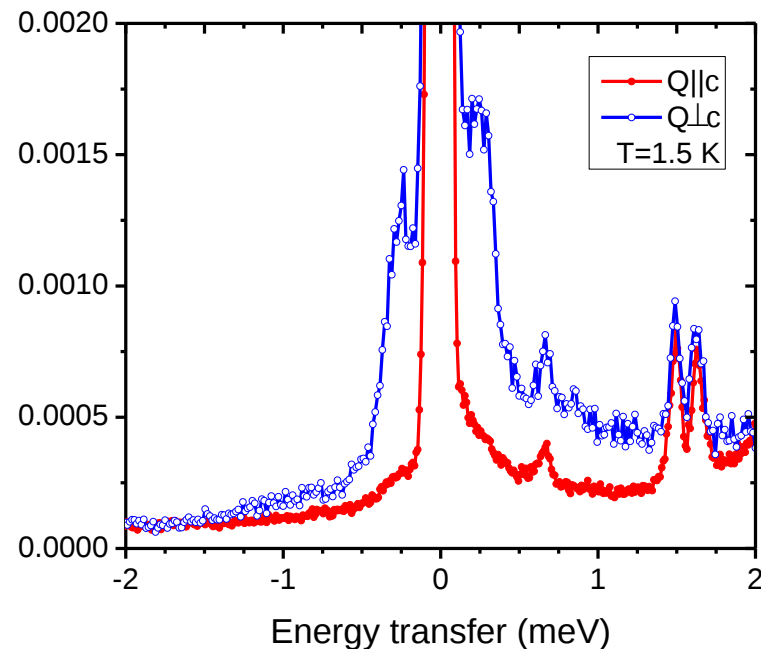
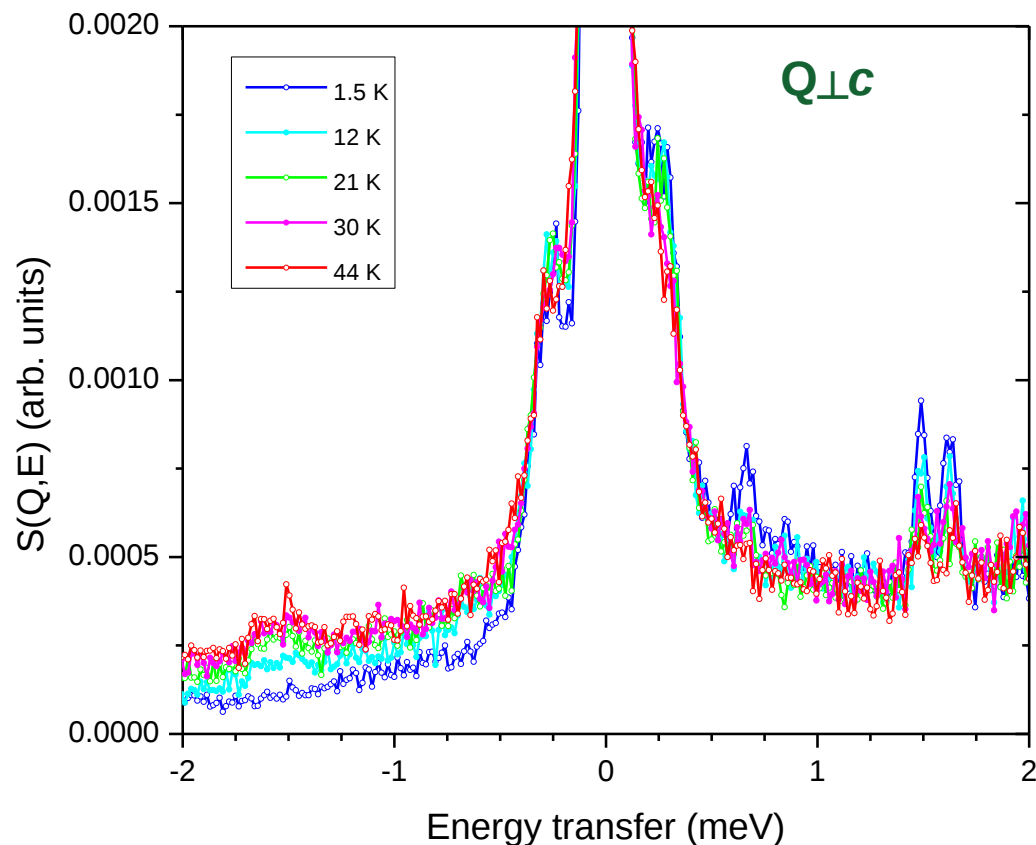
[1] M. Falk, Spectrochim. Acta A40 (1984) 43.



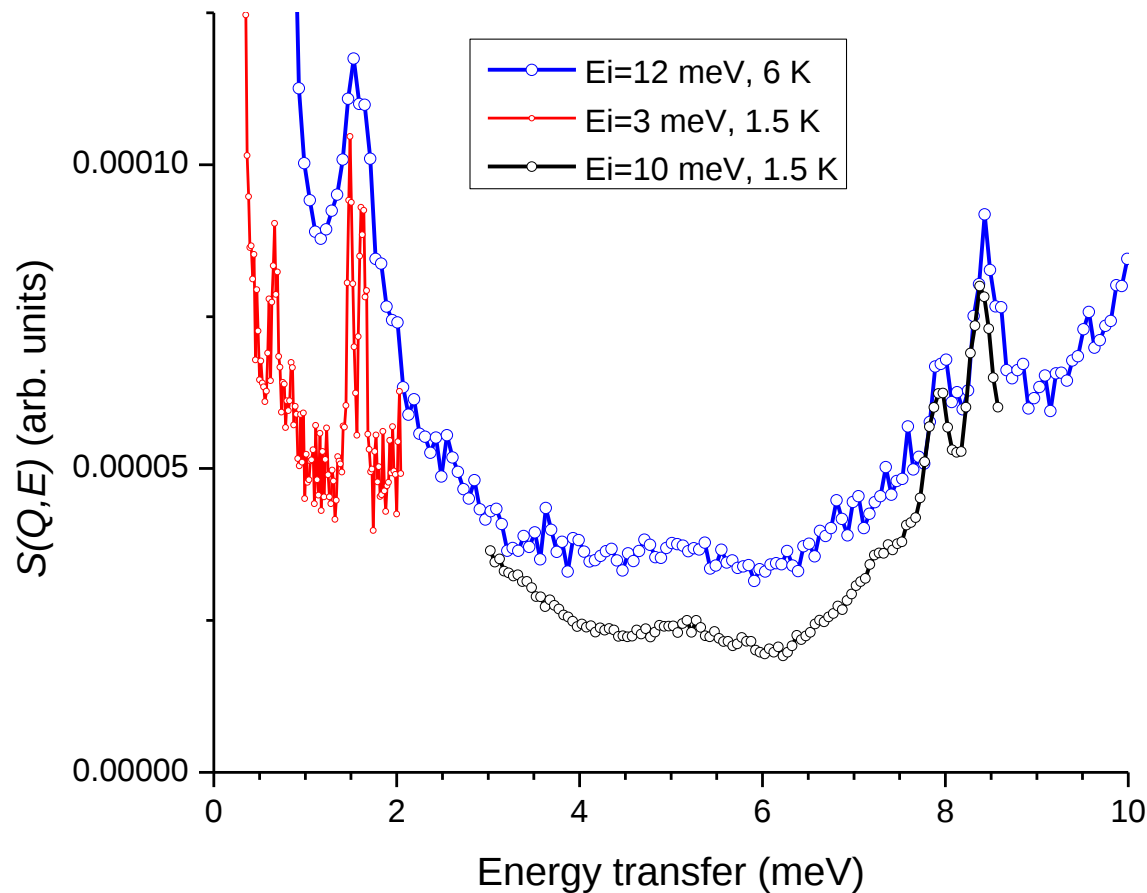
INS spectra of water in beryl, $E_i=25$ meV, Q parallel to the c -axis.



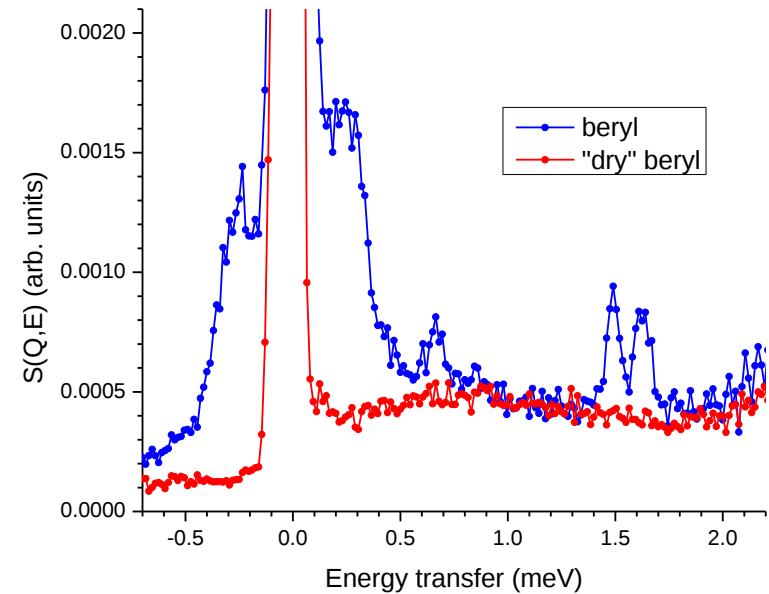
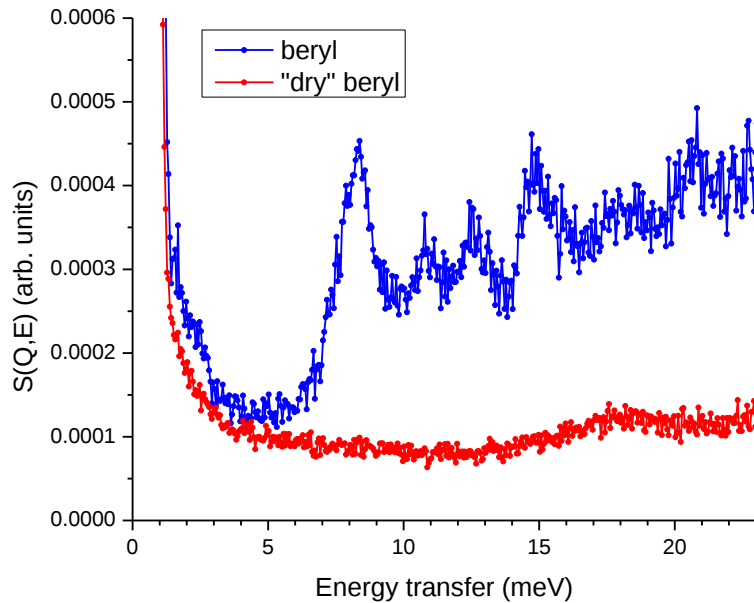
INS spectra of beryl, $E_i=25$ meV, Q perpendicular to the c -axis. The intensity of the peaks at 8.4, 12.7 and 14.7 meV strongly decrease with temperature increase.



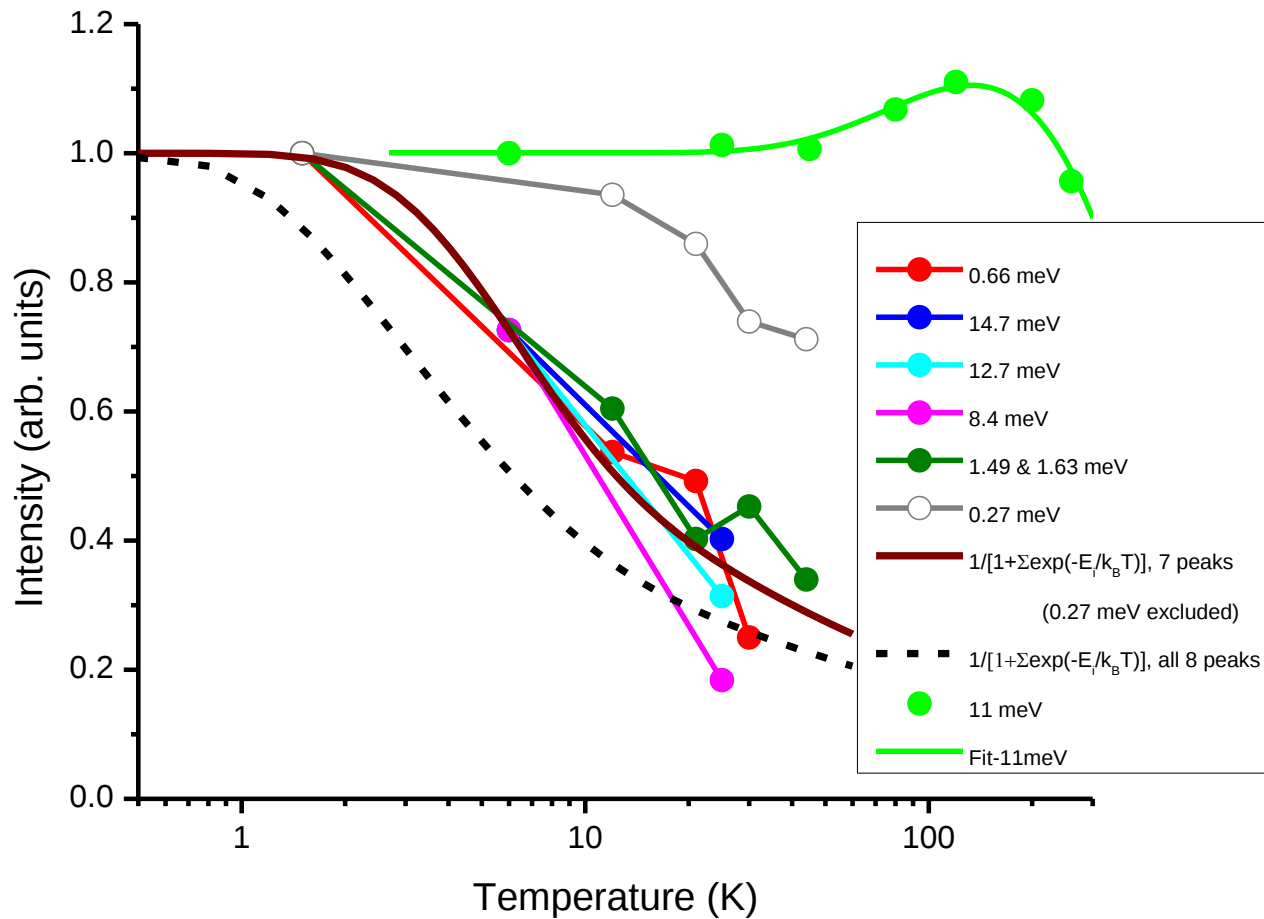
INS spectra of beryl at different temperatures measured with $E_i=3$ meV at CNCS, Q perpendicular to the c -axis (left figure). The intensity of the peaks at 0.27, 0.66, 1.49 and 1.63 meV in energy loss site strongly decrease with temperature increase. The peaks at 0.66, 1.49 and 1.63 meV are isotropic (right figure)



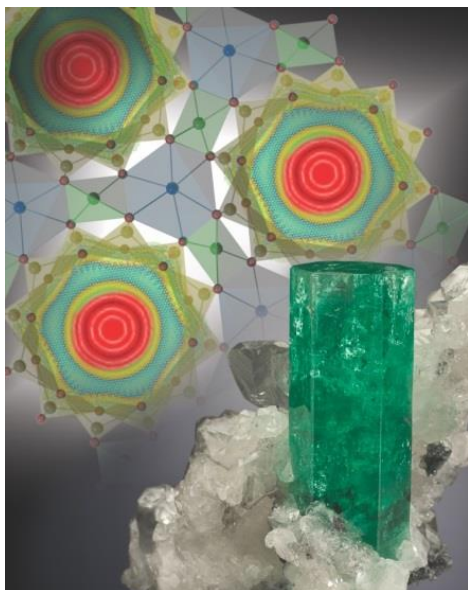
INS spectra of beryl for Q perpendicular to c -axis measured at SEQUOIA with $E_i = 12$ meV, and at CNCS with $E_i = 10$ meV, showed that the peak at 8.4 meV is split (7.9 and 8.4 meV).



INS spectra of beryl and dehydrated (“dry”) beryl measured at $T=1.5$ K with $Q \perp c$ -axis using CNCS spectrometer with $E_i=28$ and 3 meV (left and right, respectively). The spectra of the “dry” beryl show small intensity (due to smaller neutron scattering cross-sections of atoms in beryl compared to hydrogen) with very smooth featureless behavior, probably due to overlapping of many possible modes in this energy range.

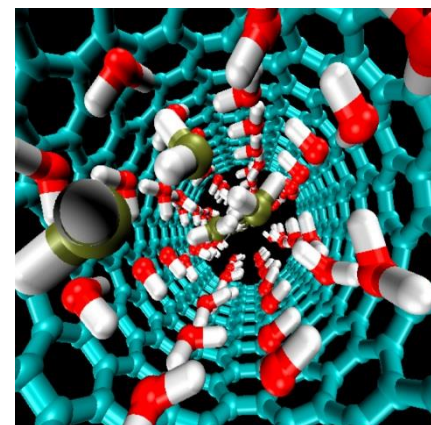


The reduced intensities of the tunneling peaks at energies $E_i=0.27, 0.66, 1.49 \text{ \& } 1.63, 7.9 \text{ \& } 8.4, 12.7$ and 14.7 meV for $Q \perp c$ -axis of beryl (except for the peak at 0.27 meV) follow the temperature dependence for transitions from the lower to higher split ground states, $I(T) \propto 1/\left\{1 + \sum_{i=1}^7 \exp\left(-\frac{E_i}{k_B T}\right)\right\}$. The intensity of the translational (vibrational) mode at 11 meV ($Q \parallel c$ -axis) is shown for comparison. The peaks' intensities are normalized to be 1 at $T=0$ K.



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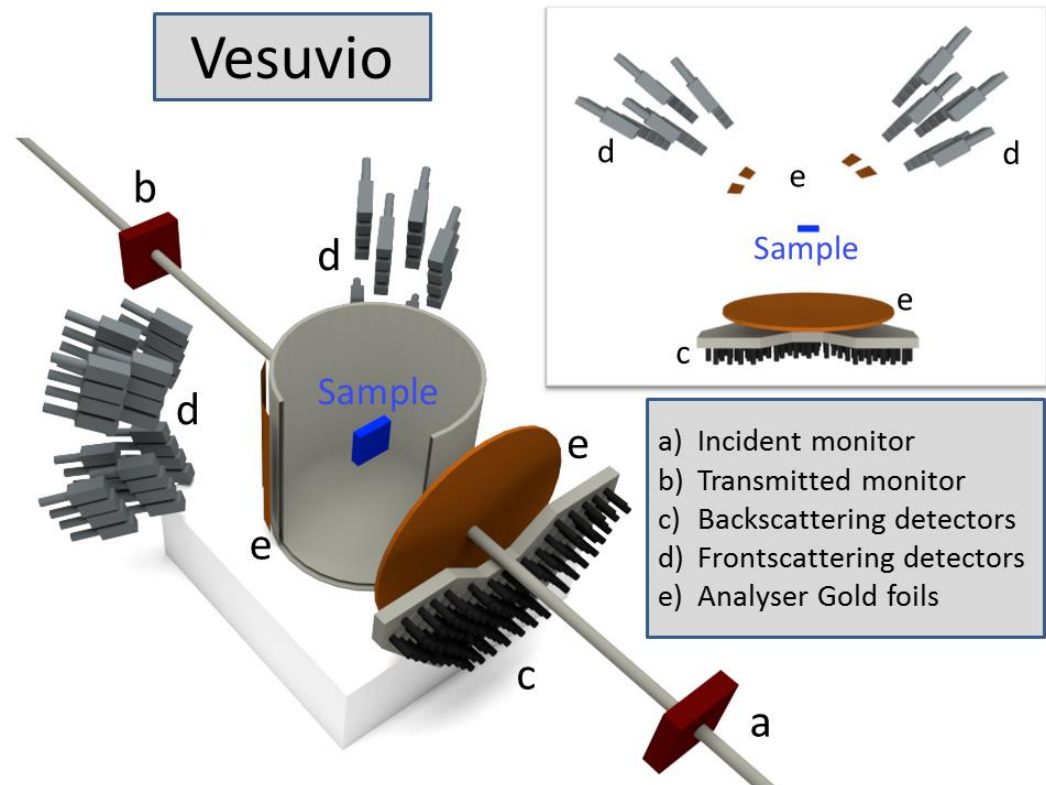
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Deep Inelastic Neutron Scattering

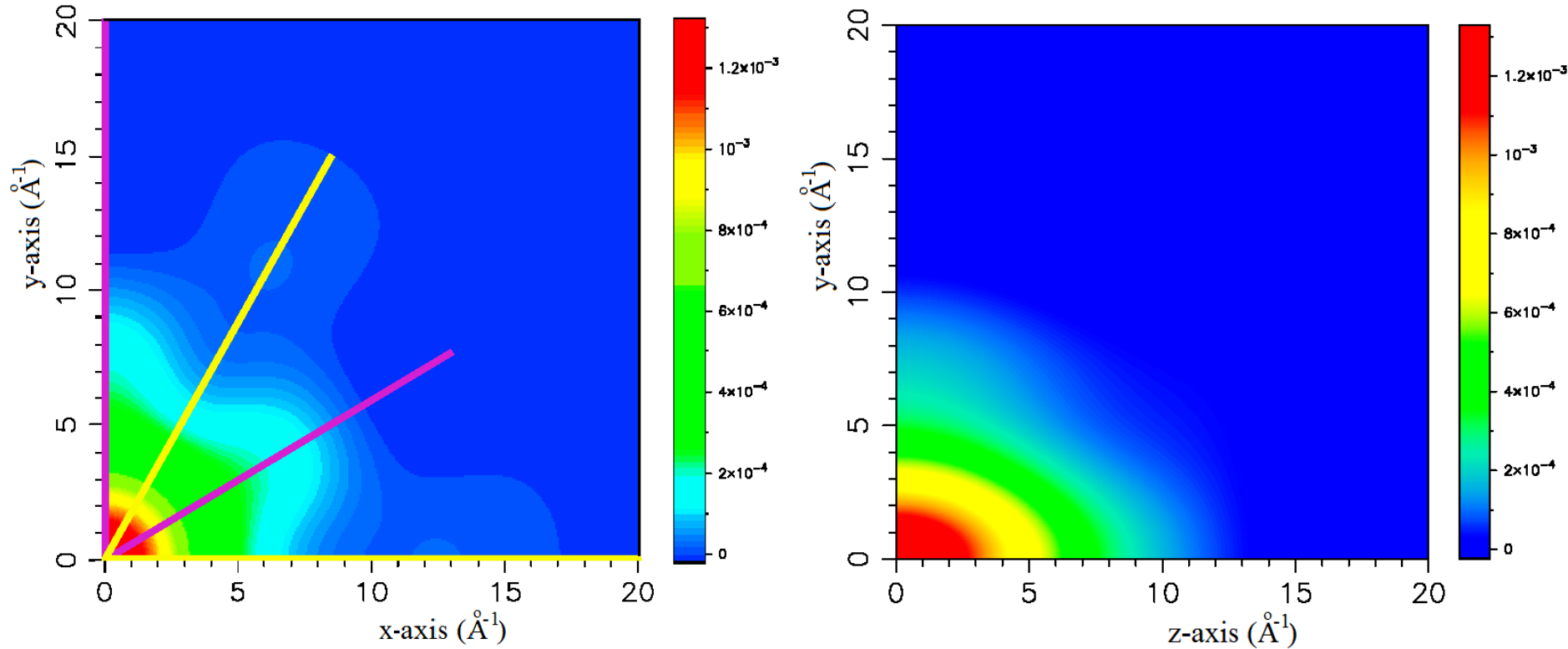
The momentum distribution $n(p)$ of the particles can be measured by Deep Inelastic Neutron Scattering (DINS). DINS is inelastic neutron scattering in the limit of large momentum transfer, Q ($\sim 30\text{-}100 \text{ \AA}^{-1}$). $S(Q, E)$ in this limit takes the form:

$$S(Q, E) = \int n(p) \delta \left(E - \frac{\hbar^2 Q^2}{2M} - \frac{\hbar Q p}{M} \right) dp$$



Analyzing energy: 4906 meV (Au)

DINS measurements were done at 4.3 K for two crystal orientations, with c-axis along and perpendicular (c-axis vertical) to the incoming neutrons. The spectra were transferred to the 3D momentum distribution of water protons in beryl $n(p)$ by simultaneous fit of all data using the Hermite polynomials of the 8 order (George Reiter, Un. Houston, TX).



Projection of water proton momentum distribution $n(p)$ onto the xy- and yz-planes.

The values of $n(p)$ broadening are: $\sigma_x=3.66$, $\sigma_y=3.61$ and $\sigma_z=4.98 \text{ \AA}^{-1}$, therefore the kinetic energy of the proton, $\langle E_K \rangle = \frac{\hbar^2}{2M} (\sigma_x^2 + \sigma_y^2 + \sigma_z^2) = 106 \text{ meV}$, is much smaller than has been observed in bulk water ($\sim 150 \text{ meV}$).

The water proton kinetic energy can be estimated by using semi-empirical approach in quasi-harmonic approximation (Moreh and Nemirovsky, *J. Chem. Phys.* 133 (2010) 084506)

$$E_K = S_t \int_0^{v_t} \alpha_t g_t(v) dv + S_r \int_{v_{r1}}^{v_{r2}} \alpha_r g_r(v) dv + \sum_{j=1}^3 S_j \frac{1}{2} \left(\frac{h\nu_j}{e^{h\nu_j/kT} - 1} + \frac{h\nu_j}{2} \right)$$

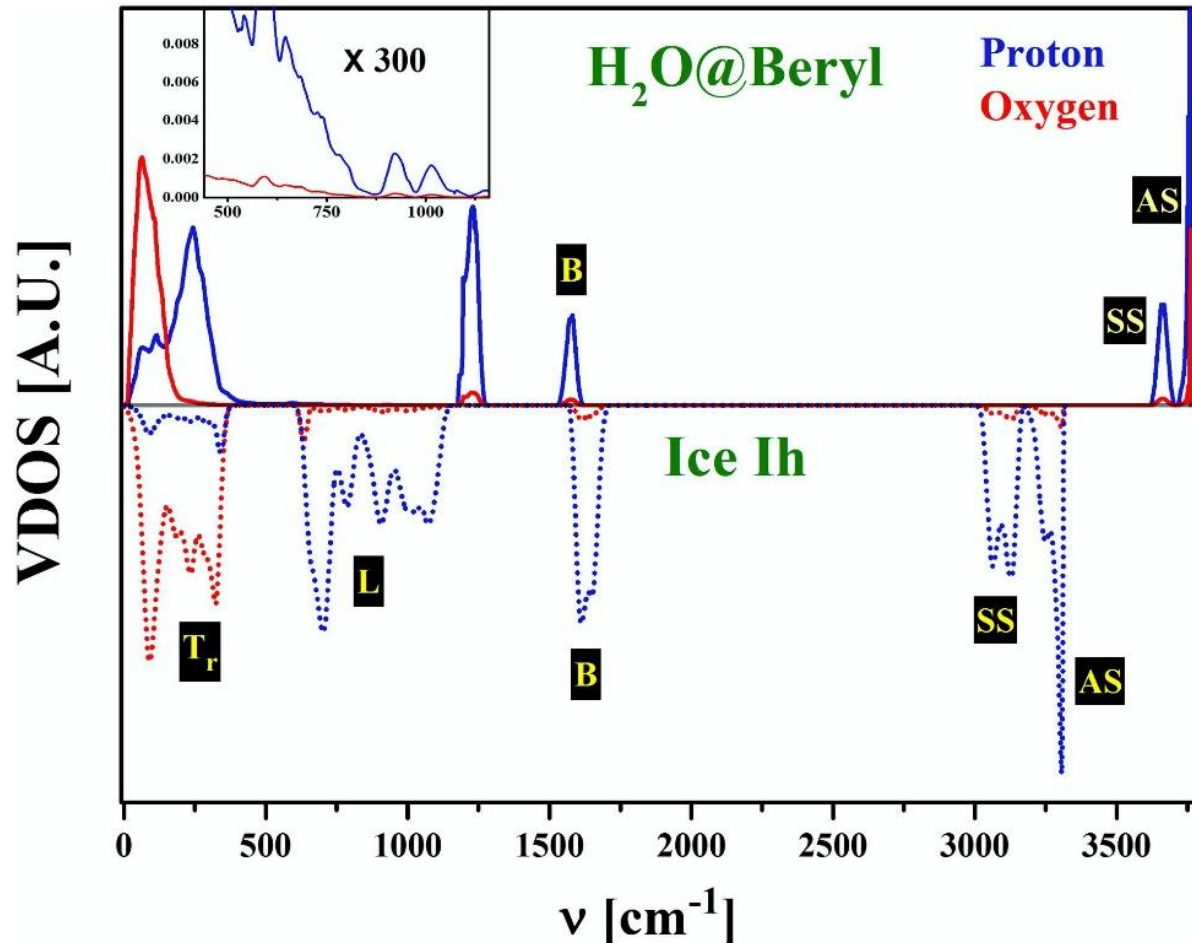
$$\alpha_i = \left(\frac{h\nu_i}{e^{h\nu_i/kT} - 1} + \frac{h\nu_i}{2} \right)$$

and S_j is the kinetic energy fraction of atom j , which for water proton is about 0.47. In the case of beryl, using only the intramolecular vibrations of water obtained from INS spectra ($\nu_1 = \nu_3 = 465$ meV, and $\nu_2 = 197.5$ meV) and omitting the translational and librational vibrations of water the estimated

$$E_K = 0.47 \times (2 \times 465/2 + 197.5/2) = 133 \text{ meV},$$

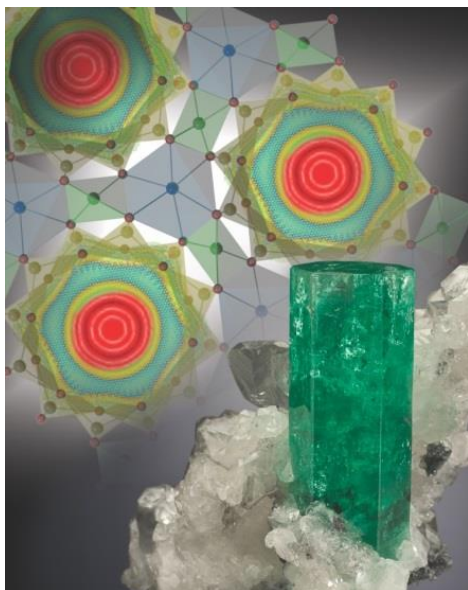
which is larger, than the observed (DINS) experimental value, $E_K=106$ meV.

Quantum behavior of water nano-confined in beryl



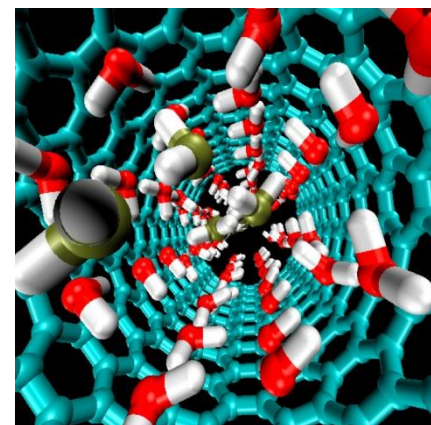
The water proton E_K in beryl at 5 K was obtained by simulating the partial VDOS from DFT calculations. The result, $E_K=104.4$ meV, is in remarkable agreement with the DINS measured value of 106 meV. The calculation indicates that the vibrational states of the proton of the nano-confined water molecule distribute much differently than in ordinary H_2O phases, most probably due to coupling with lattice modes of the hosting beryl nano-cage.

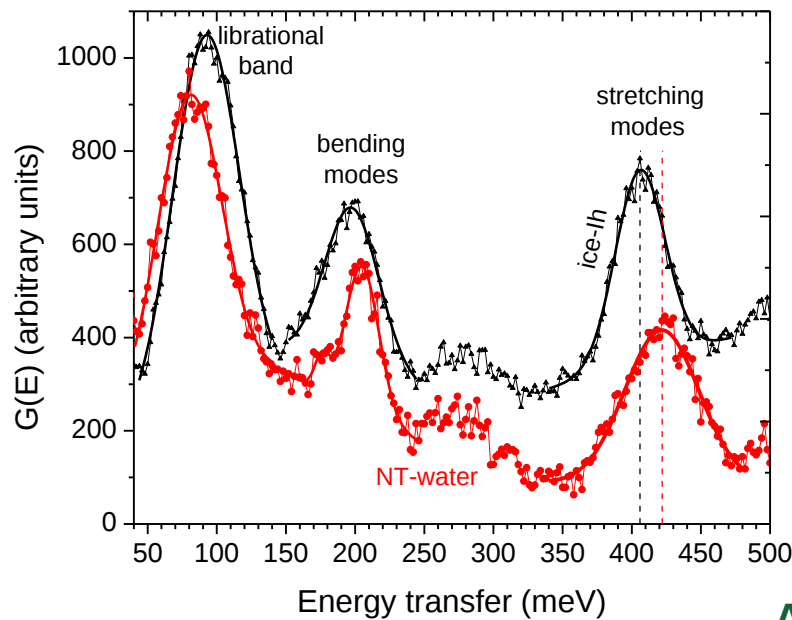
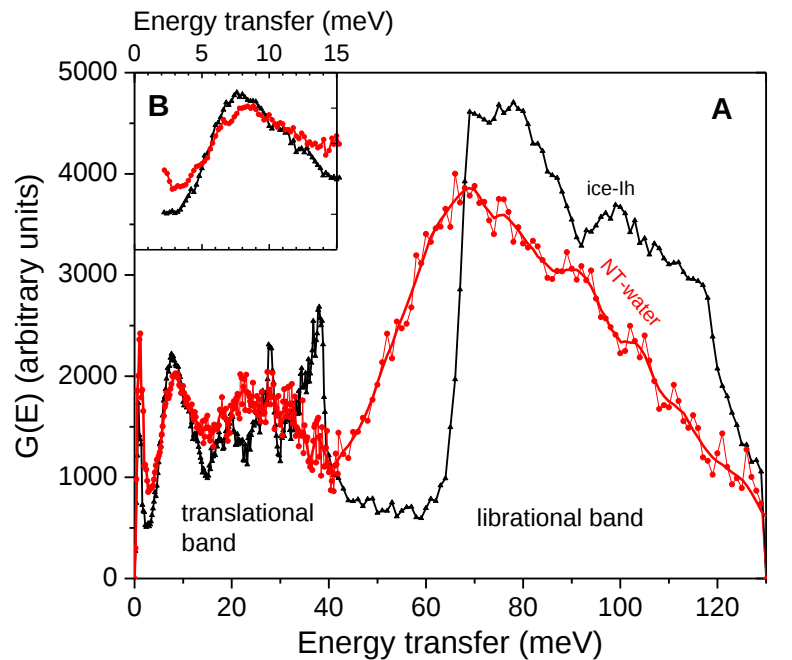
Y. Finkelstein *et al.*, J. Chem. Phys. 146 (2017) 124307



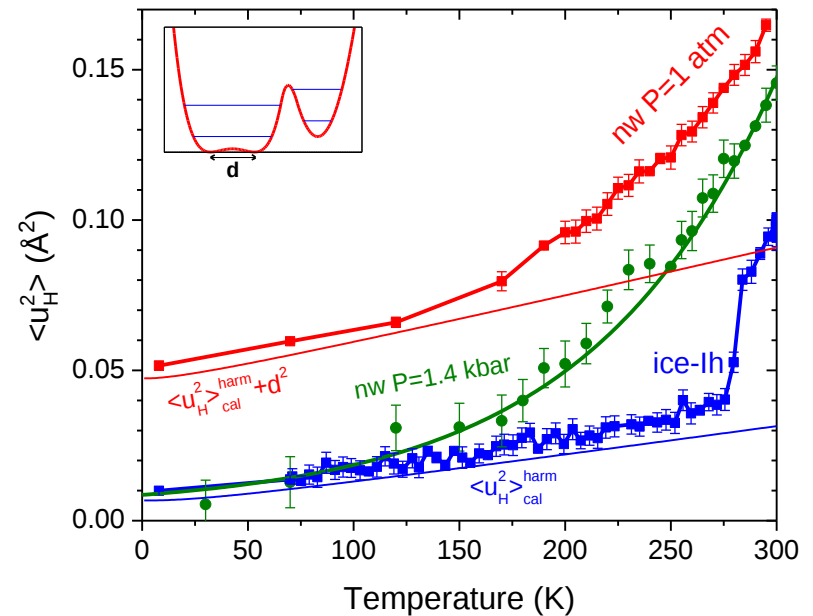
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Water in SWNT

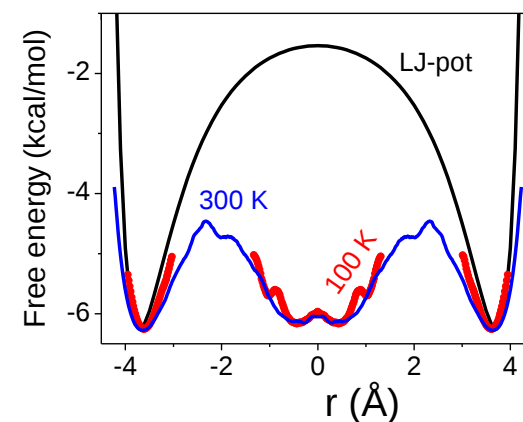
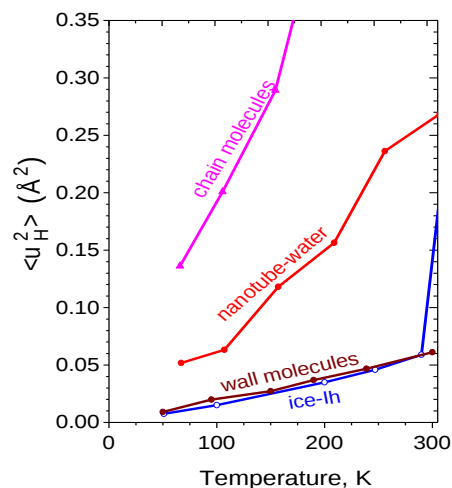
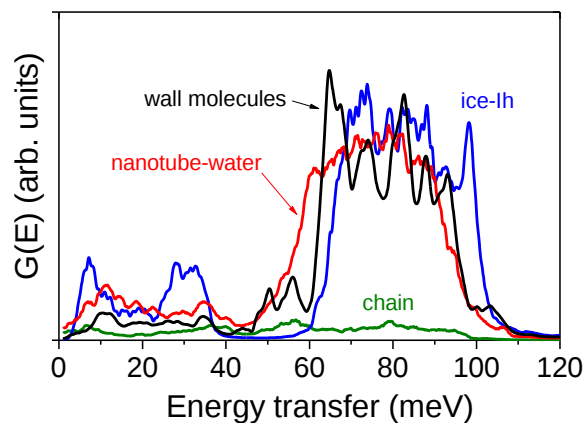
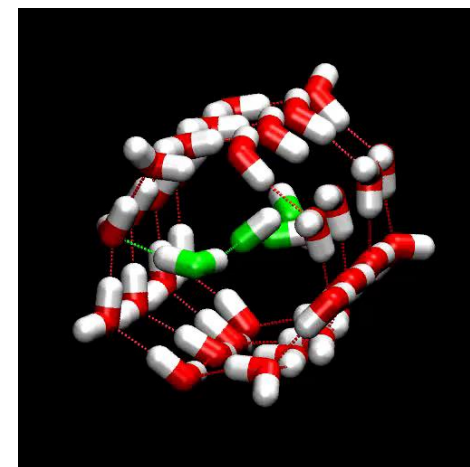
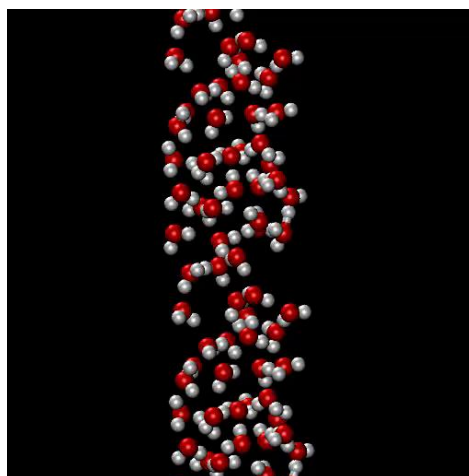
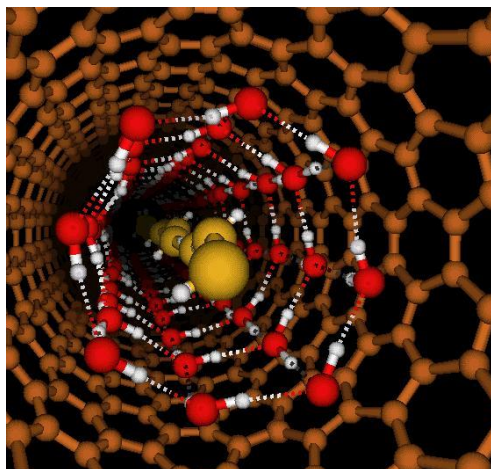


MSD of water protons in SWNT compared to ice-Ih obtained from QENS.

Vibrational spectra of water in SWNT compared to the spectrum of ice-Ih.

A.I. Kolesnikov *et al.*, Phys. Rev. Lett. 93 (2004) 035503

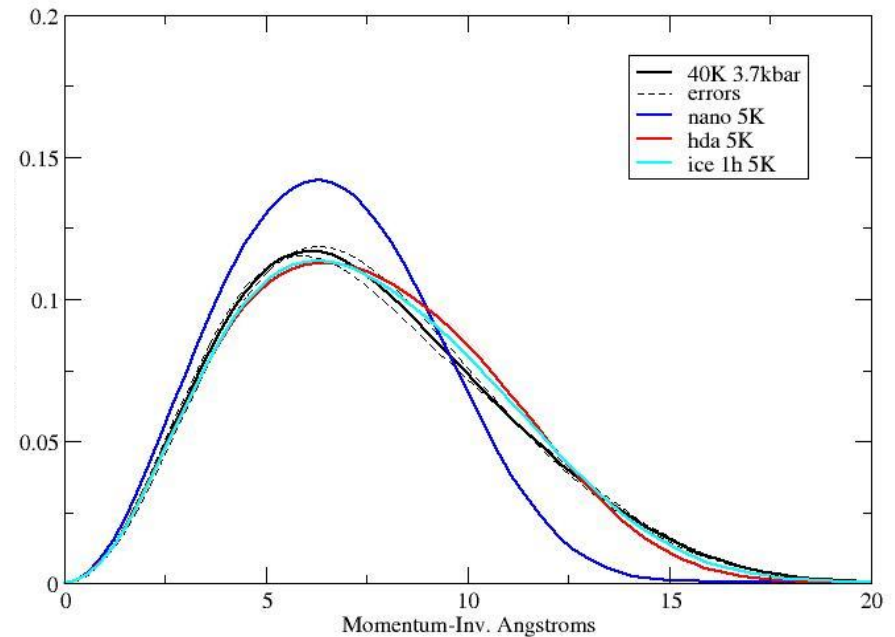
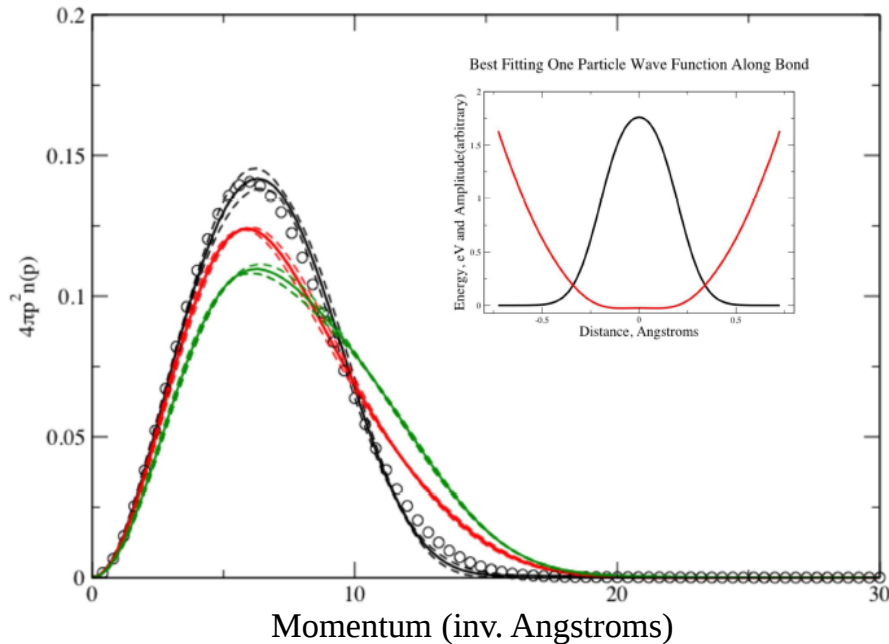
Water in SWNT



MD calculations (Christian Burnham) with TTM2-F polarizable flexible water model (uses smeared charges and dipoles) [1]. MD simulations consist of 40 Å long carbon nanotube in periodic boundary conditions that interacts with water via the LJ potential [2].

[1] Burnham & Xantheas, J. Chem. Phys. (2002); [2] Walther *et al.*, J. Phys. Chem. (2001)

Deep-Inelastic Neutron Scattering



Momentum distributions for NT-water at 268 K (green) and 5 K (black), and **ice-Ih at 269 K (red)**. The circles are a fit to a model in which the water proton is delocalized in a double well potential. The potential (red) and wave-function (black) are shown in the inset.

$\langle E_K \rangle = 104$ meV for NT-water at 5 K,
 $\langle E_K \rangle = 146-152$ meV for bulk water.

G. Reiter *et al.*, Phys. Rev. Lett. 97 (2006) 247801

The momentum distribution of the protons in NT-water at $P=1$ bar ($T=5$ K) and $P=3.7$ kbar ($T=40$ K) compared with bulk ice-Ih and hda-ice ($T=5$ K) at $P=1$ bar.

Summary

- INS spectra of beryl at low temperature show 8 peaks at energies 0.25–15 meV, and the intensity of these peaks strongly decreases with increasing temperature. The data can be explained by transitions between the split ground states of confined water due to its tunneling between the 6-fold equivalent positions across the crystal channels.
- DINS data show that water protons exhibit 6-fold momentum distribution across the beryl channel, with additional maxima due to proton coherent delocalization (tunneling). The obtained proton kinetic energy, $E_K=106$ meV, is much smaller than in bulk water (~150 meV), in spite of the observed highest possible O-H stretching modes.
- INS and QENS measurements of water in SWNT showed very soft dynamics of nanotube-water. MD simulations identified an ice-wall plus water-chain structure at low temperatures. The soft dynamics of nanotube-water arise mainly from the drastic change in hydrogen-bond connectivity of the central water-chain with an average coordination number less than 2 (compared to 4 in bulk ice).
- DINS showed that the quantum state of the nanotube-water protons at low temperature is qualitatively different from that of bulk water, and gave very small value for kinetic energy, $E_K=104$ meV.

Acknowledgements

George Ehlers **ORNL**

Eugene Mamontov

Andrey Podlesnyak

Narayani Choudhury **Un. Washington, Bothell, WA**

Jean-Marc Zanotti **IPNS, ANL**

Chun-Keung Loong

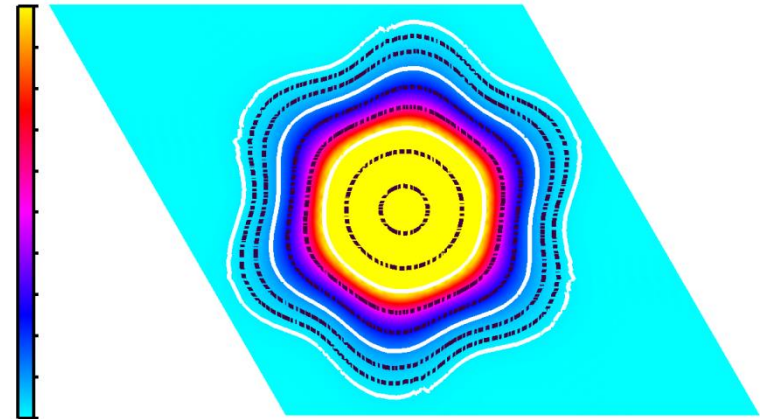
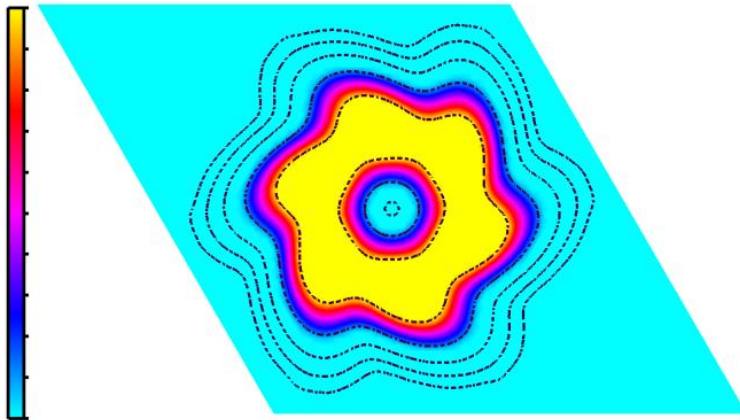
Pappannan Thiyagarajan

Alexander P. Moravsky **MER Corporation, Tucson, AZ**

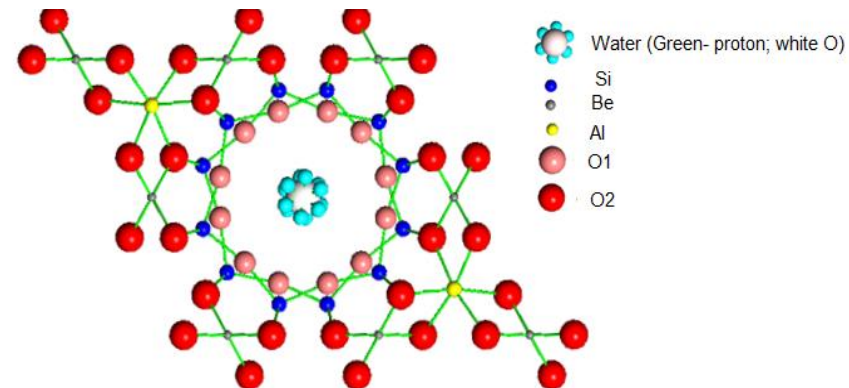
Christian J. Burnham **University of Utah, UT**

J. Mayers **ISIS, RAL, UK**

DFT simulations (Narayani Choudhury, Un. Washington) reveals proton delocalization in beryl, with a lateral spread of about 1.2 Å, consistent with the experiments. Lattice excitations of the beryl cage couple with local modes of water and yield adiabatic coincidence states in the vicinity of which the energy barriers for water rotation are significantly lowered, and the confined water molecules exhibit quantum tunneling.



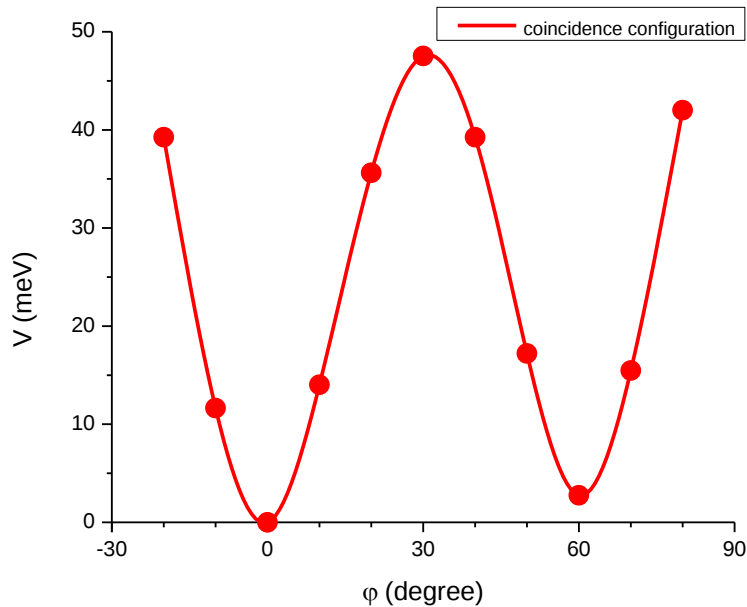
The calculated water charge density slices along the ab -plane at the positions of the proton (left) and oxygen (right) atoms. The contours represent isoenergy lines; the bounding box is $(-0.18, 0.18)$ along the a - and b -axes. The min/max charge (abs.val.) in the color-bars are $0.0224/0.0985 \text{ Ha}/\text{\AA}^3$ (a), and $0.0224/ 13.6546 \text{ Ha}/\text{\AA}^3$ (b).



The DFT calculated structure of beryl well agrees with the experiment of Gatta *et al.*, Am. Mineral. 91 (2006) 29.

$$H = -\frac{\hbar^2}{2I} \frac{\partial^2}{\partial \varphi^2} + V(\varphi) \quad V(\varphi) = \frac{V_6}{2} (1 - \cos 6\varphi)$$

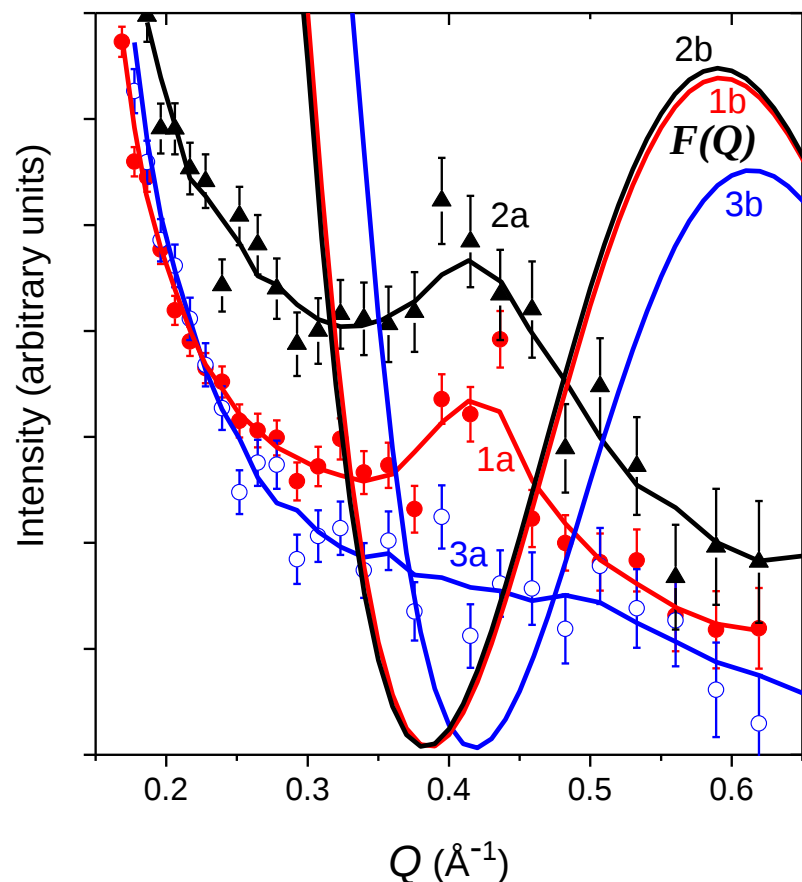
$$H_{nm} = \frac{\hbar^2 n^2}{2I} \delta_{n,m} + \frac{V_6}{4} \{2\delta_{n,m} - \delta_{n-m+6,0} - \delta_{n-m-6,0}\}, \quad n, m = 0, \pm 1, \pm 2, \dots$$



m	E_m (meV) $V_6=48$ meV	ΔE (meV) $V_6=48$ meV	Transitions ($m=0$) $\rightarrow$ (m')
0	21.42		
1, 5	24.16	2.74	0 $\rightarrow$ 1, 0 $\rightarrow$ 5
2, 4	31.81	10.39	0 $\rightarrow$ 2, 0 $\rightarrow$ 4
3	38.73	17.31	0 $\rightarrow$ 3

Water enters SWNT by exposing them to water vapor at 110°C

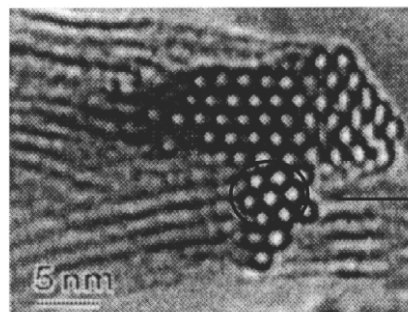
Low-angle neutron diffraction: $I(Q) \sim S(Q) \cdot F(Q)$. Here, $S(Q)$ consists of a Bragg reflection at $0.41 \text{ \AA}^{-1}$ from the (01) planes of the 2D hexagonal lattice of SWNT crystalline bundles.



1 – Dry SWNT; 2 – SWNT & H₂O
3 – SWNT & D₂O

1698

J.-L. Sauvajol et al. / Carbon 40 (2002) 1697–1714



TEM picture of SWNT bundles

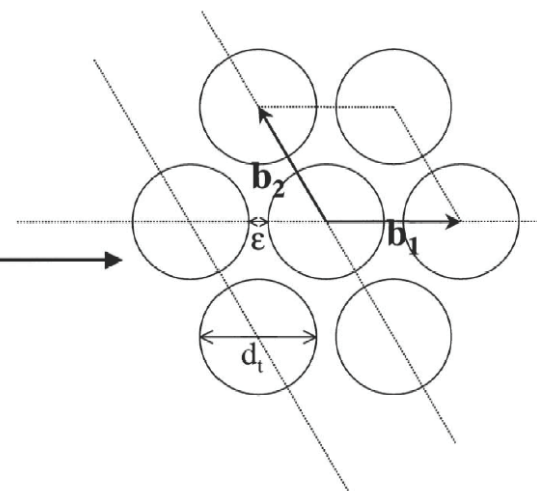
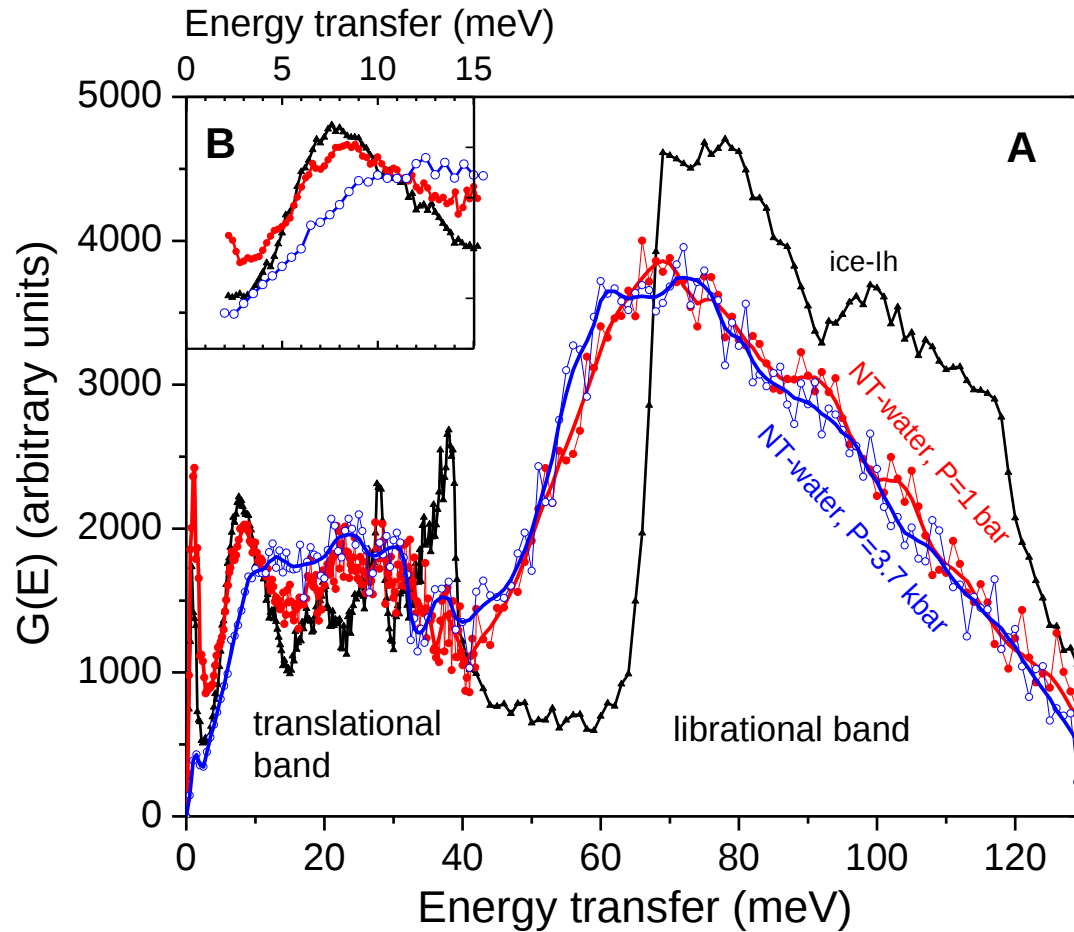


Fig. 1. Left: transmission electron microscopy observation of crystalline packing of SWNT in bundles (from Ref. [10]). Right: sketch of the 2D hexagonal lattice.

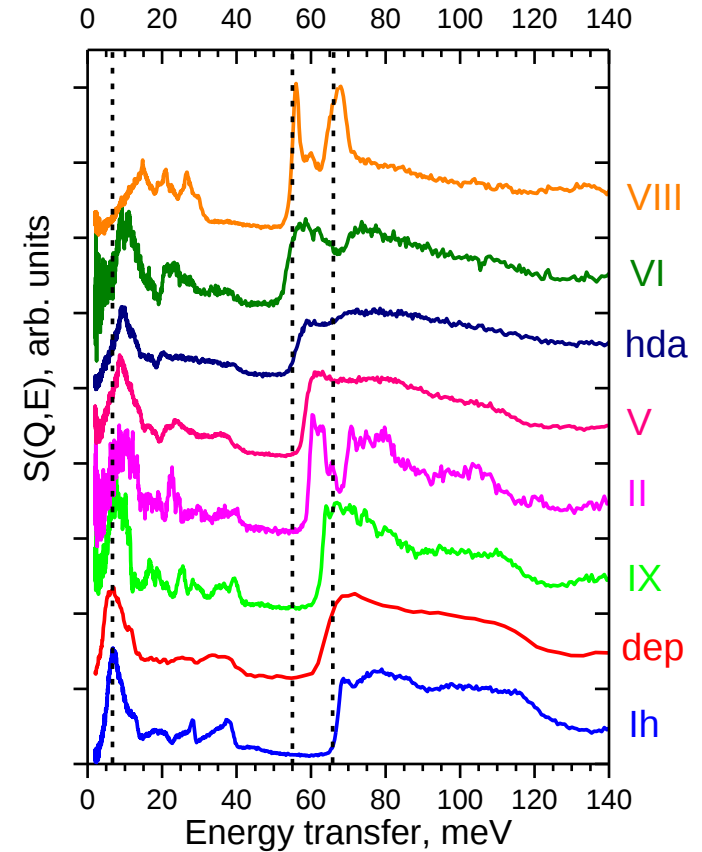
SWNT ($D \approx 14 \pm 1 \text{ \AA}$, $l \sim 10 \text{ \mu m}$) was characterized by HRTEM, TEM, SEM, Raman, ND.

To fill SWNT with water, the dry SWNT was exposed to water vapor at 110°C. The excess water in the exterior of the nanotubes was evaporated at 35°C. An optimal filling, H₂O/SWNT mass ratio, was found to be 11%.

Water in SWNT and pressure effect

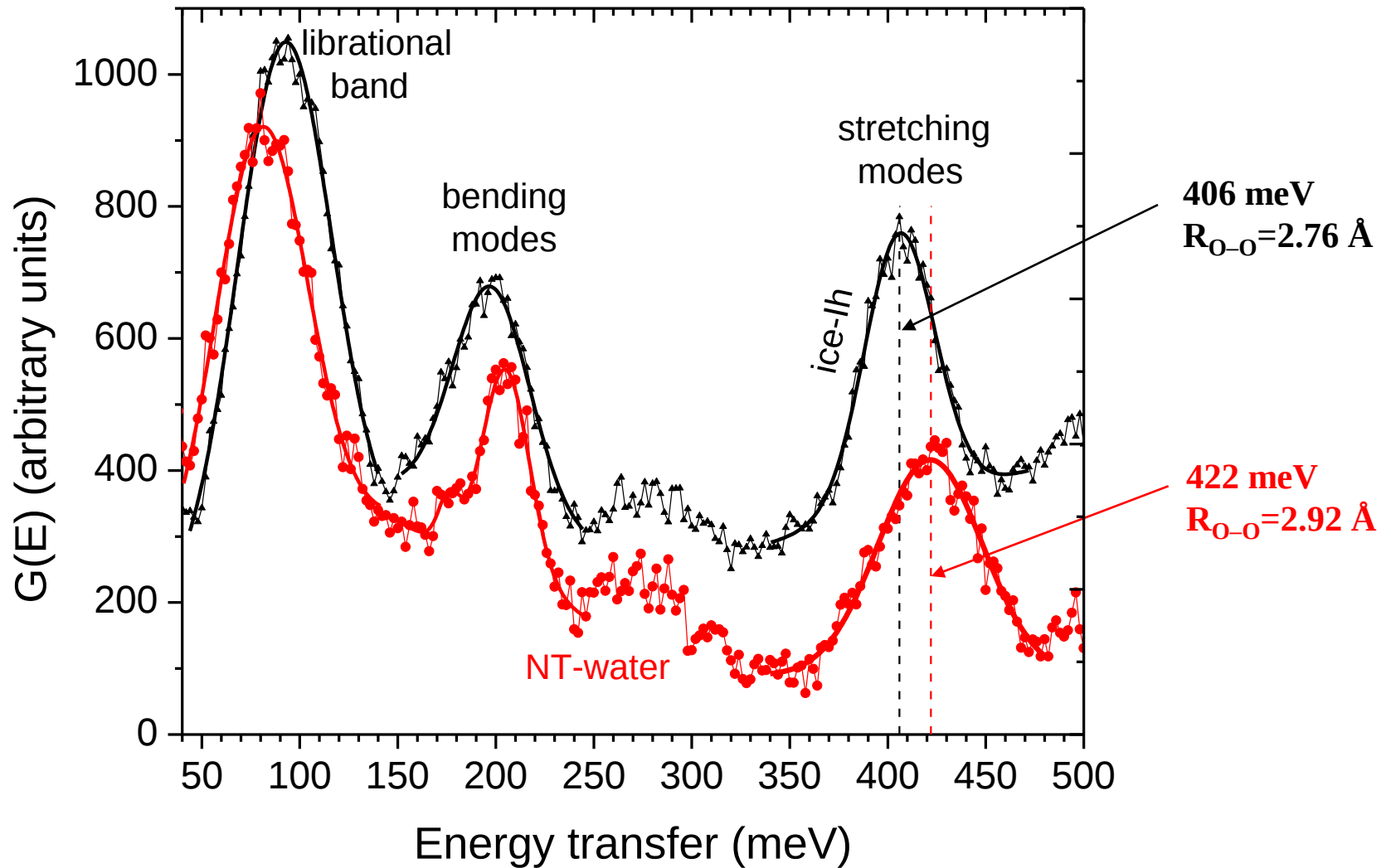


Pressure effect

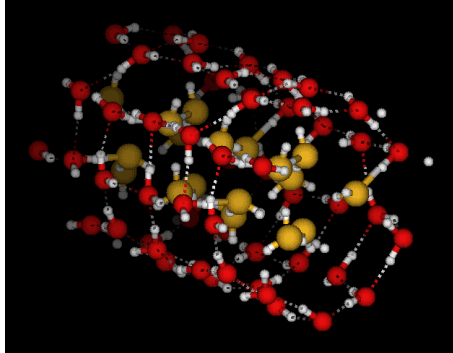
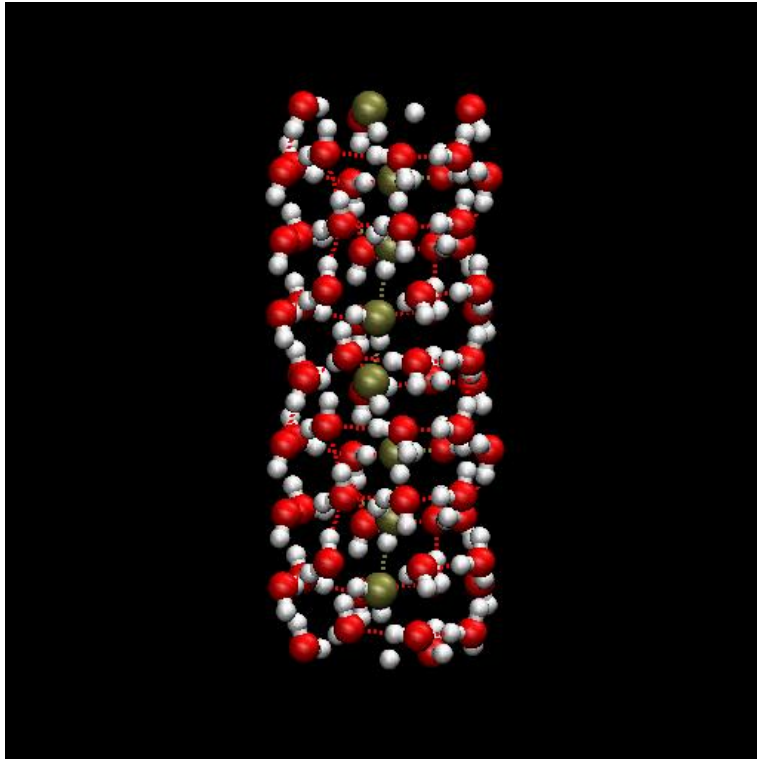


INS spectra of different phases of bulk ice

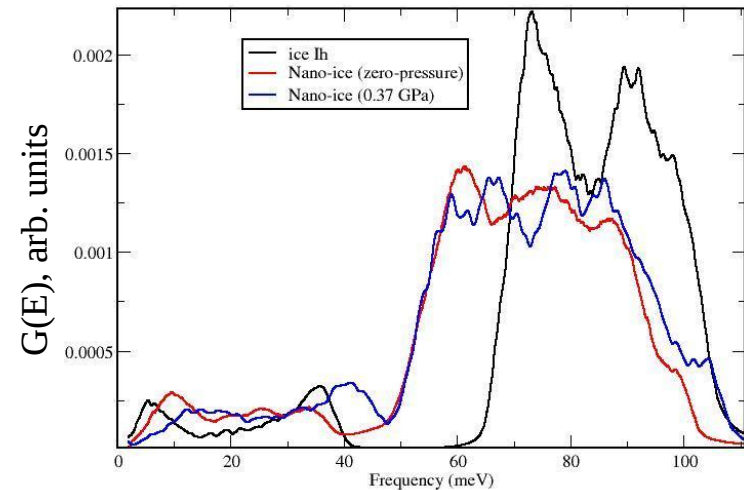
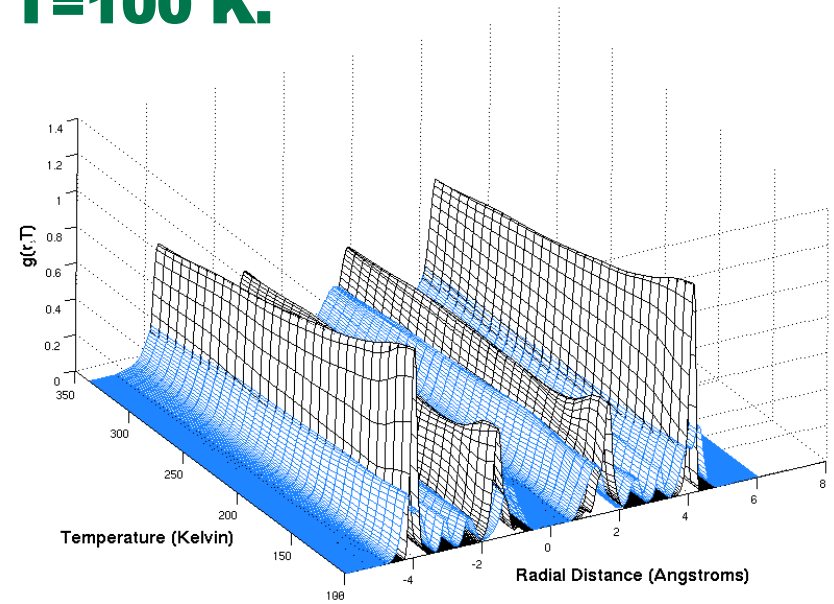
A Weak Hydrogen-Bond Network in Nanotube-Water



Pressure effect on the calculated structure and vibrational spectra of nanotube-water at T=100 K.



Proposed
structure of
high pressure
nanotube-water



Calculated vibrational spectra for
nanotube-water at P=1 bar and
P=3.7 kbar, compared to ice-Ih

Inelastic neutron scattering (INS) is one of a preferred method to study the dynamics of hydrogen containing materials owing to the extraordinarily large scattering cross-section of hydrogen and the straightforward comparison of the experimental data with the results of model simulations.

$$S(Q, E) = \sum_{l,k} \sigma_H^{inc} \exp(-2W(Q)) \left(\frac{\hbar Q^2}{6M} \right)^k \int d\omega_1 \dots d\omega_k \frac{G(\omega_1) \dots G(\omega_k)}{\omega_1 \dots \omega_k (k-l)!!} \prod_{i=l+1}^k [n_B(\omega_i, T) + 1] \prod_{j=1}^l n_B(\omega_j, T) \delta(E - \sum_{i=l+1}^k \hbar\omega_i + \sum_{j=1}^l \hbar\omega_j)$$

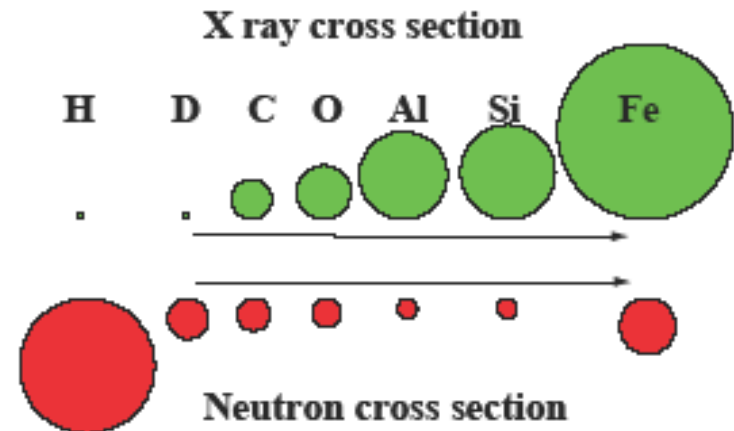
$$S_{1ph}(Q, \omega) = \sigma_H^{inc} \frac{\hbar Q^2}{2M} \exp(-2W(Q)) \frac{G(\omega)}{\omega} [n_B(\omega, T) + 1]$$

where

$$W = \frac{1}{2} \langle (\mathbf{Q} \cdot \mathbf{u})^2 \rangle = \frac{\hbar Q^2}{2M} \int \frac{G(\omega)}{\omega} [2n_B(\omega, T) + 1] d\omega$$

and $G(\omega)$ can be calculated as

$$G(\omega) = \sum_{j,\mathbf{q}} \frac{1}{3N} |\mathbf{e}(j, \mathbf{q})|^2 \delta[\omega - \omega(j, \mathbf{q})]$$



Quantum rotation of ortho- and para-water encapsulated in a fullerene cage

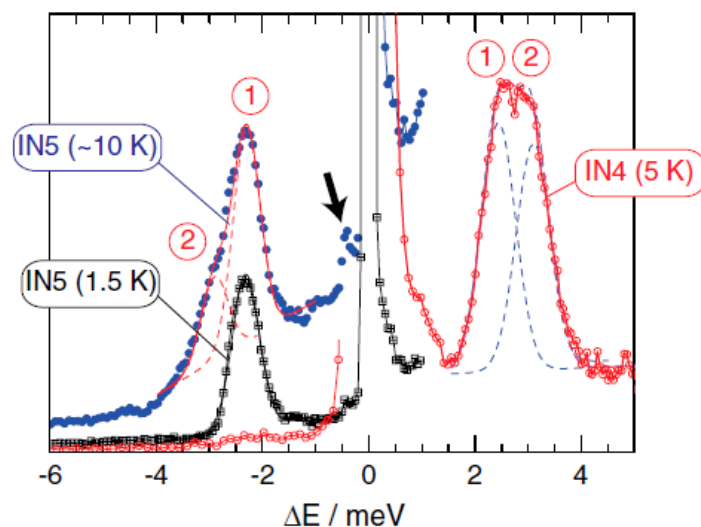
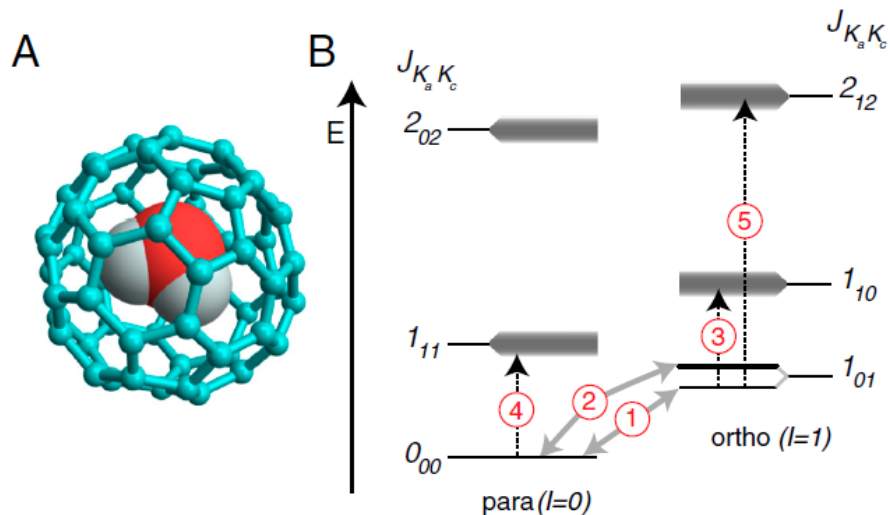
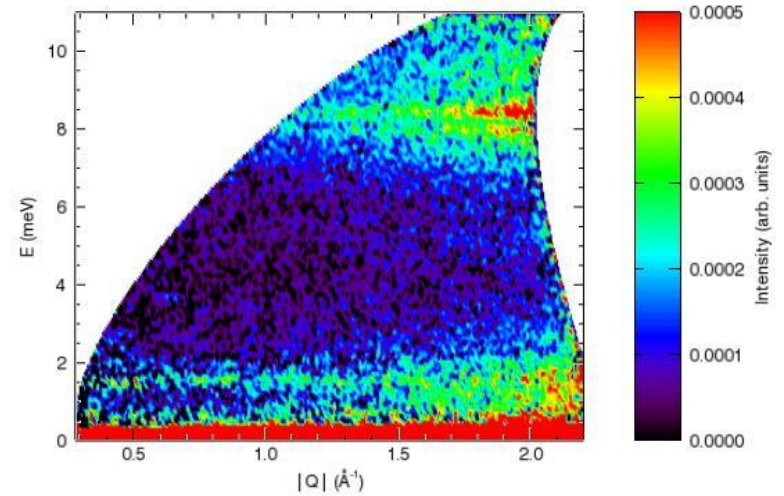
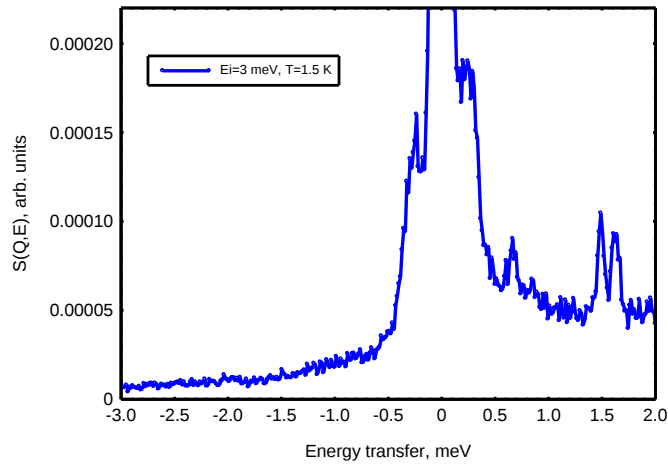
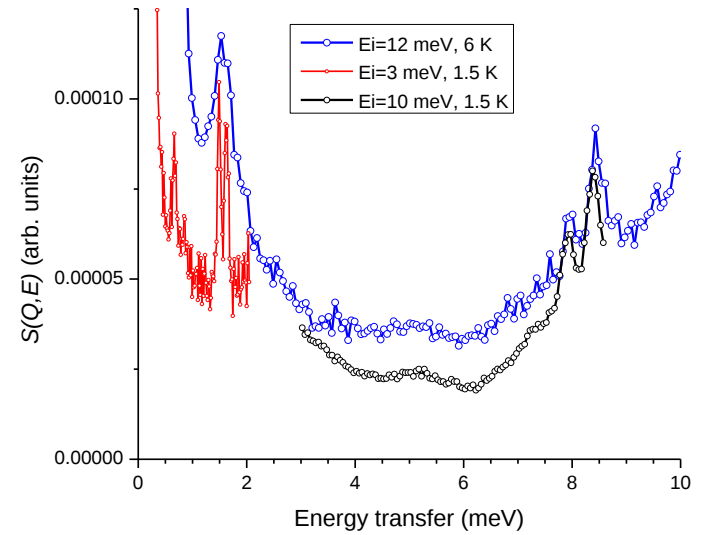
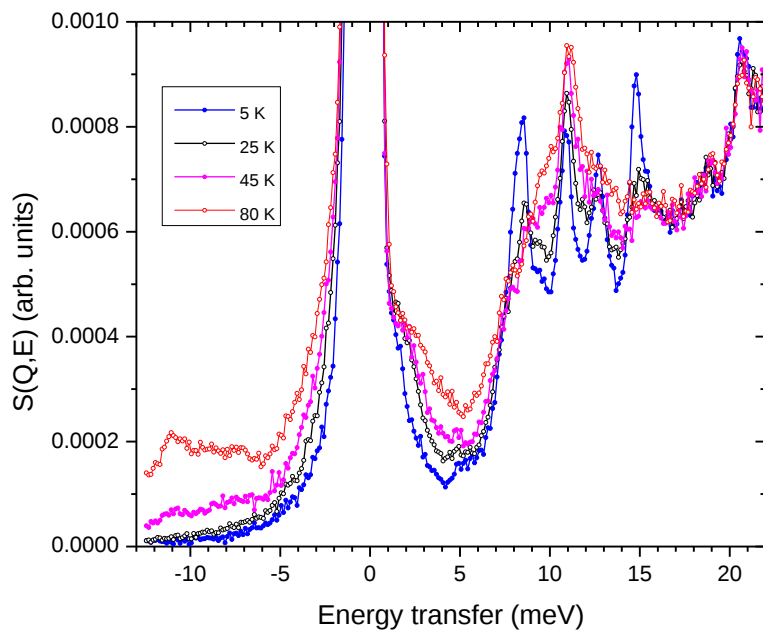


Fig. 2. INS spectra recorded on the time-of-flight spectrometers IN4 and IN5. The 10 K spectrum is an average over 5, 10, and 15 K data. The dashed lines show the best fit components where the transitions are labeled according to Fig. 1B.

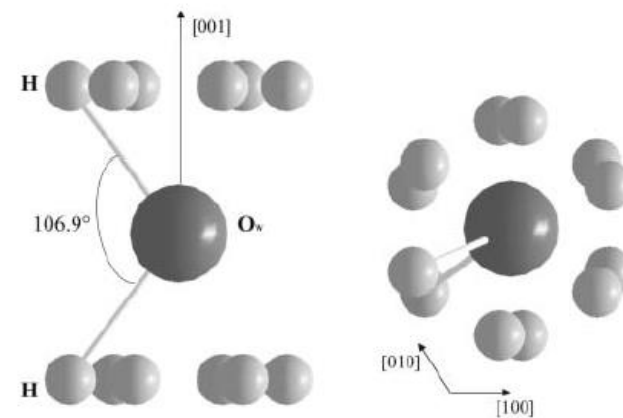
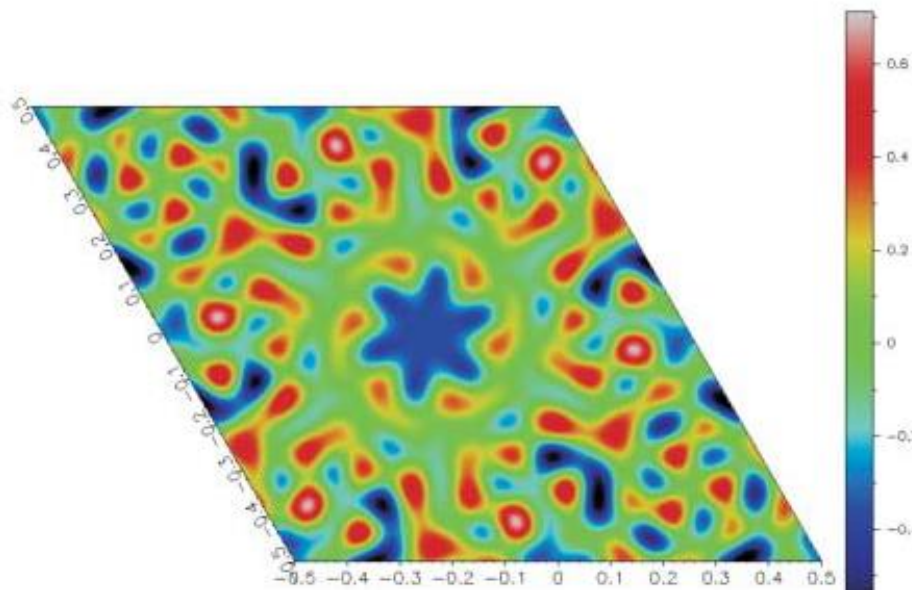
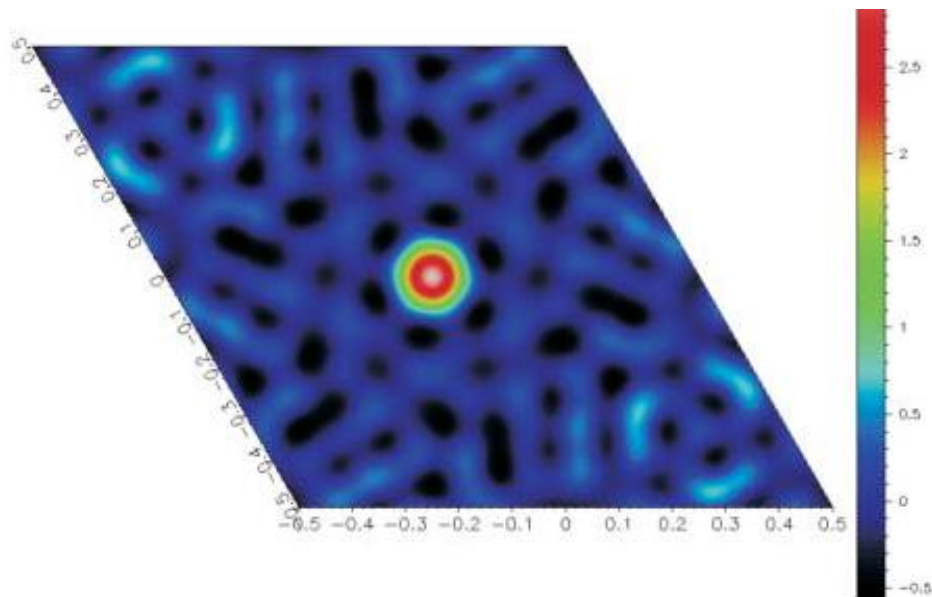
Carlo Beduz^a, Marina Carravetta^b, Judy Y.-C. Chen^c, Maria Concistrè^b, Mark Denning^b, Michael Frunzi^c, Anthony J. Horsewill^d, Ole G. Johannessen^b, Ronald Lawler^e, Xuegong Lei^c, Malcolm H. Levitt^{b,1}, Yongjun Li^c, Salvatore Mamone^b, Yasujiro Murata^f, Urmaz Nagel^g, Tomoko Nishida^f, Jacques Ollivier^h, Stéphane Rols^h, Toomas Rõõm^g, Riddhiman Sarkar^b, Nicholas J. Turro^c, and Yifeng Yang^a

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PNAS 109 (2012) 12894



No para-ortho transitions of water in beryl were observed, probably water in beryl is not as free as water in fullerene.



Topological configuration of water molecules into the channel of the alkali-poor beryl, viewed down [100] (left) and down [001] (right).

G.D. Gatta *et al.*, Am. Mineral. 91 (2006) 29.

FIGURE 2. Difference-Fourier maps of the nuclear density of beryl at $z = 0.25$ (above) and $z = 0.332$ (below) after the first cycles of isotropic refinement with a channel-free structure. At $z = 0.25$ a residual peak of about $+2.9 \text{ fm}/\text{\AA}^3$ is seen lying on the sixfold axis, whereas at $z = 0.332$ there are six negative residual peaks (with a star shaped distribution) of about $-0.6 \text{ fm}/\text{\AA}^3$ around the sixfold axes. Note that the color scale is different for the two maps.

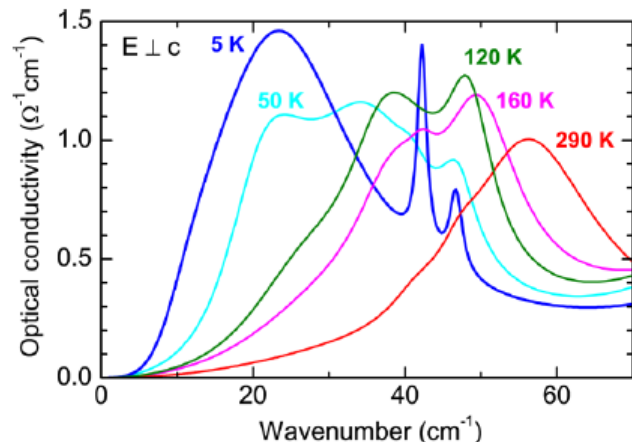


Figure 5. THz conductivity due to water absorption in beryl measured for radiation polarized perpendicular to the axis of the nanopore at different temperatures, as indicated.

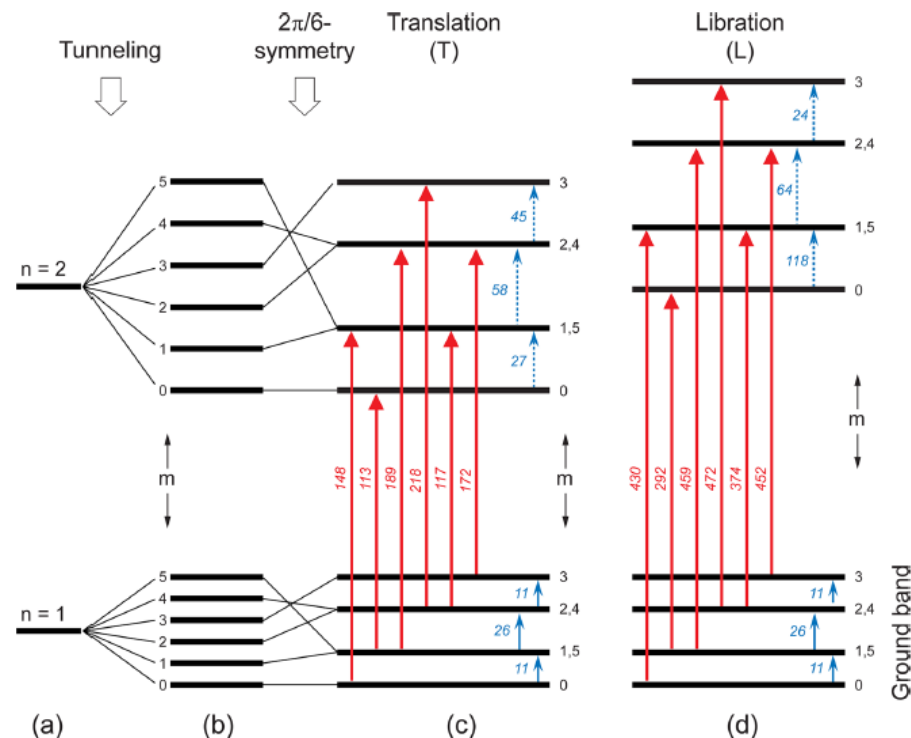
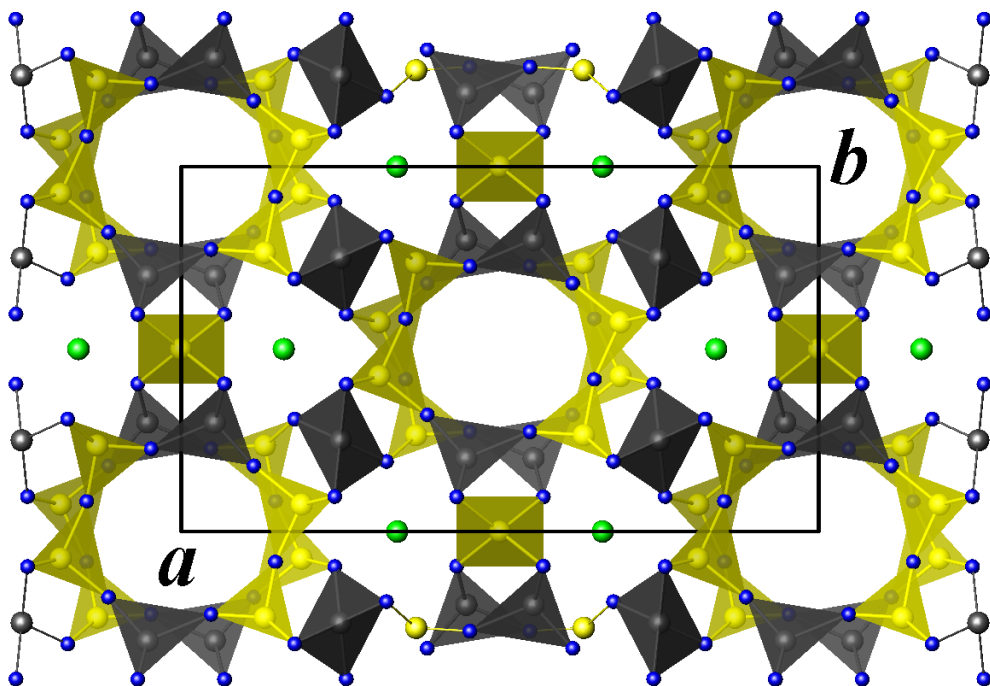


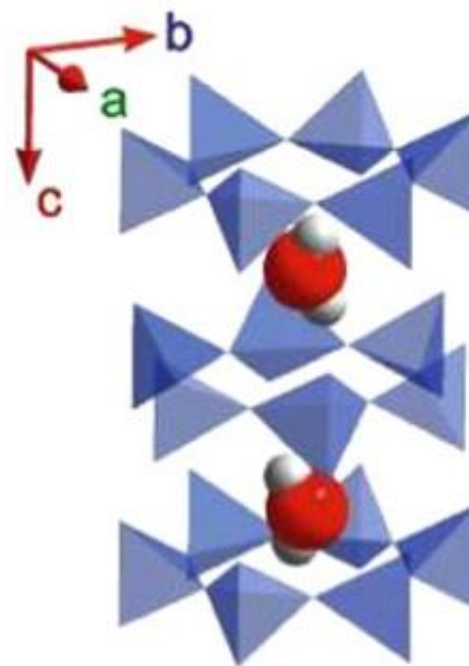
Figure 4. Scheme of vibronic energy levels of a type-I water molecule in a nanocavity of the beryl crystal lattice. The lowest-energy bands are shown. (a) Ground state ($n = 1$) and first excited in-plane vibrational state ($n = 2$) for small vibrations around a preferred direction. (b) Due to tunneling within the six-well potential, the states split according to their angular quantum number m . Two couples are degenerate, $m = 1$ and $m = -1 = 5$, and also $m = 2$ and $m = -2 = 4$ due to the six-fold symmetry. (c,d) The long red arrows indicate interband transitions, while the short blue arrows correspond to transitions that are not seen in our experiment. The selection rule $\Delta m = \pm 1$ allows only certain transitions. The dotted arrows correspond to transitions that are not seen in our experiment. The corresponding frequencies are given in units of cm^{-1} , where 100 cm^{-1} corresponds to 12 meV or 3 THz .

The tunnel splitting of water vibrational states in beryl were predicted in [1], where the authors used optical spectroscopy. The positions of the observed optical peaks assigned to intra-ground-band transitions with energies 1.33, 3.21, 5.21 and 5.82 meV disagree with our INS data, where isotropic tunneling peaks were observed at 0.66, 1.49 and 1.63 meV, and anisotropic peaks at 0.27, 7.9 & 8.4, 12.7 and 14.7 meV, which were visible only for $Q \perp c$ -axis.

[1] Gorshunov *et al.*, J. Phys. Chem. Lett. 4 (2013) 2015

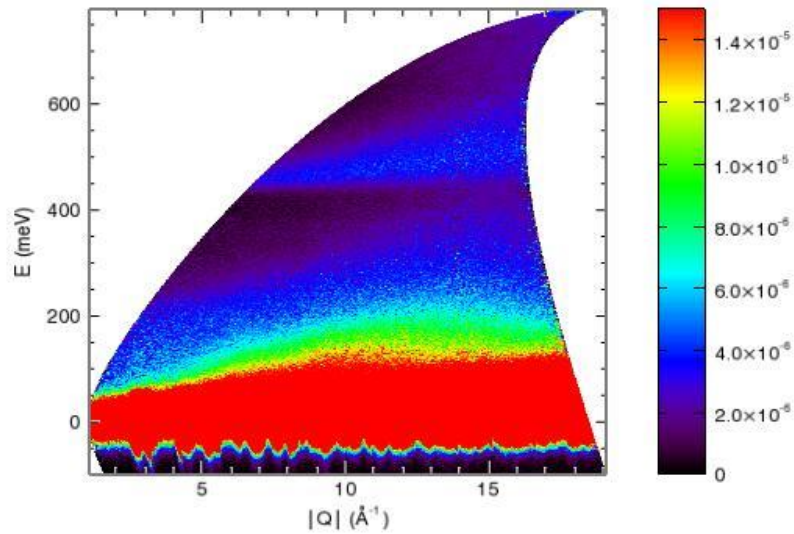


Structure of cordierite ($\text{Al}_4\text{Mg}_2\text{Si}_5\text{O}_{18}$) showing large open channels running parallel to the crystallographic c -axis (out of the figure's plane): Si, yellow; Al, blue; O, red; and Be, green.

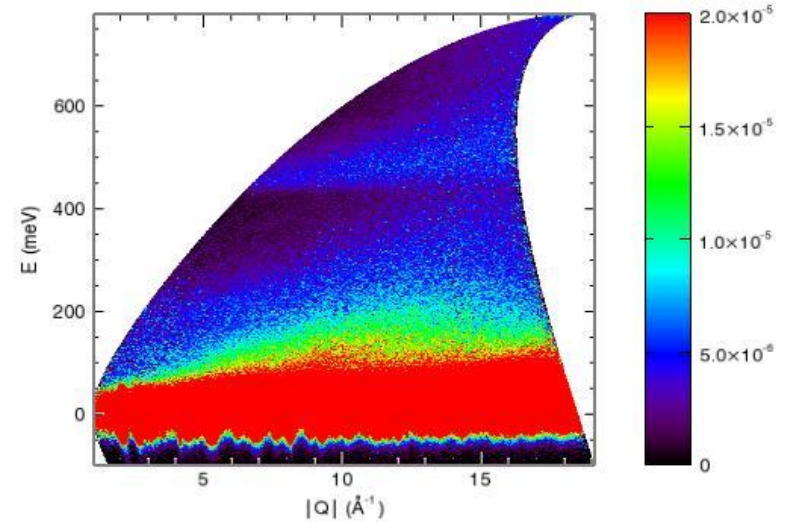


Graphical representation of the alignment of water molecules inside the channels of cordierite.

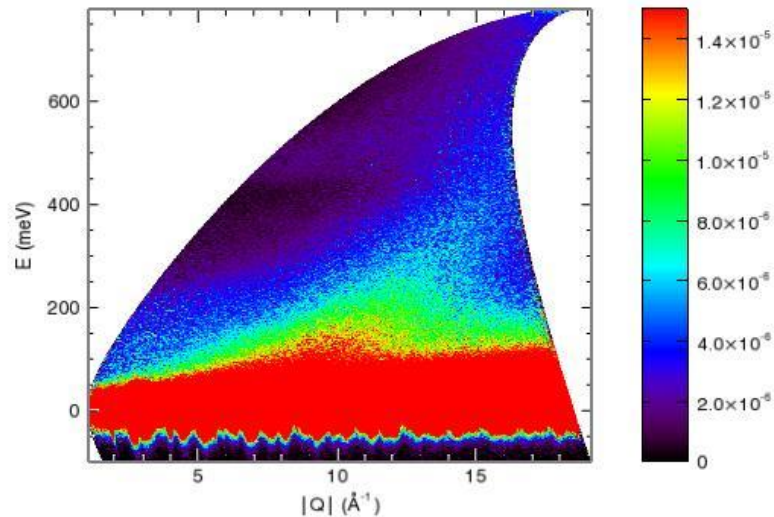
Beryl, Q||c



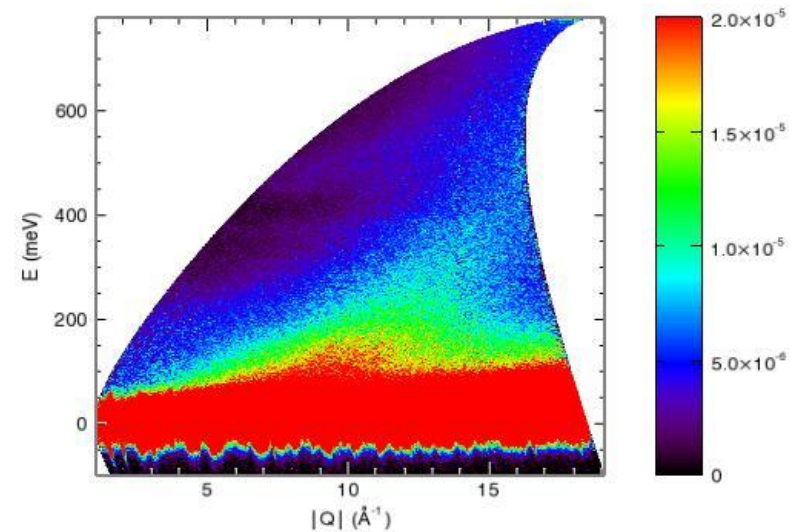
S(Q,E) spectra, SEQUOIA



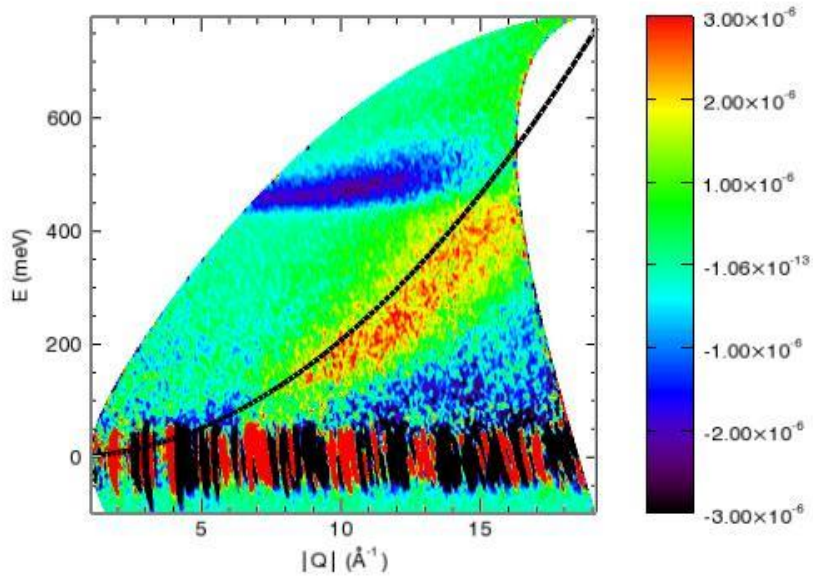
Beryl, Q⊥c



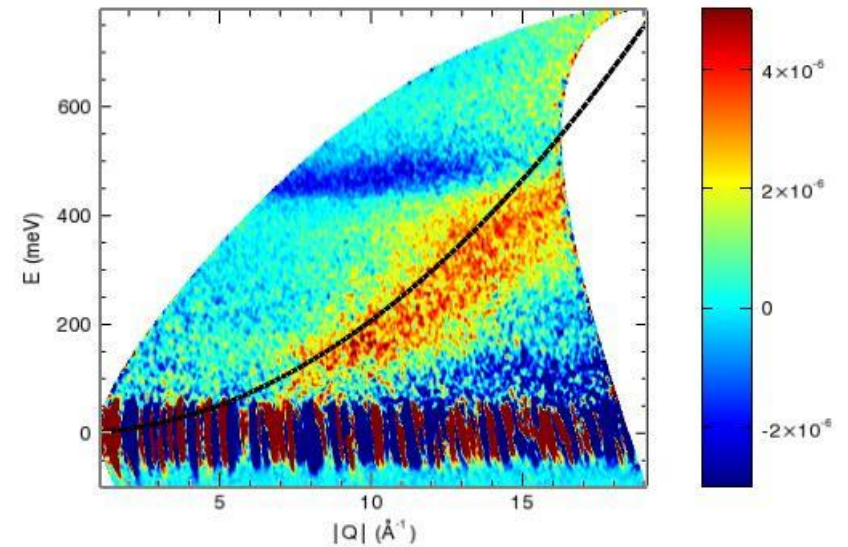
Cordierite, Q⊥c



Beryl

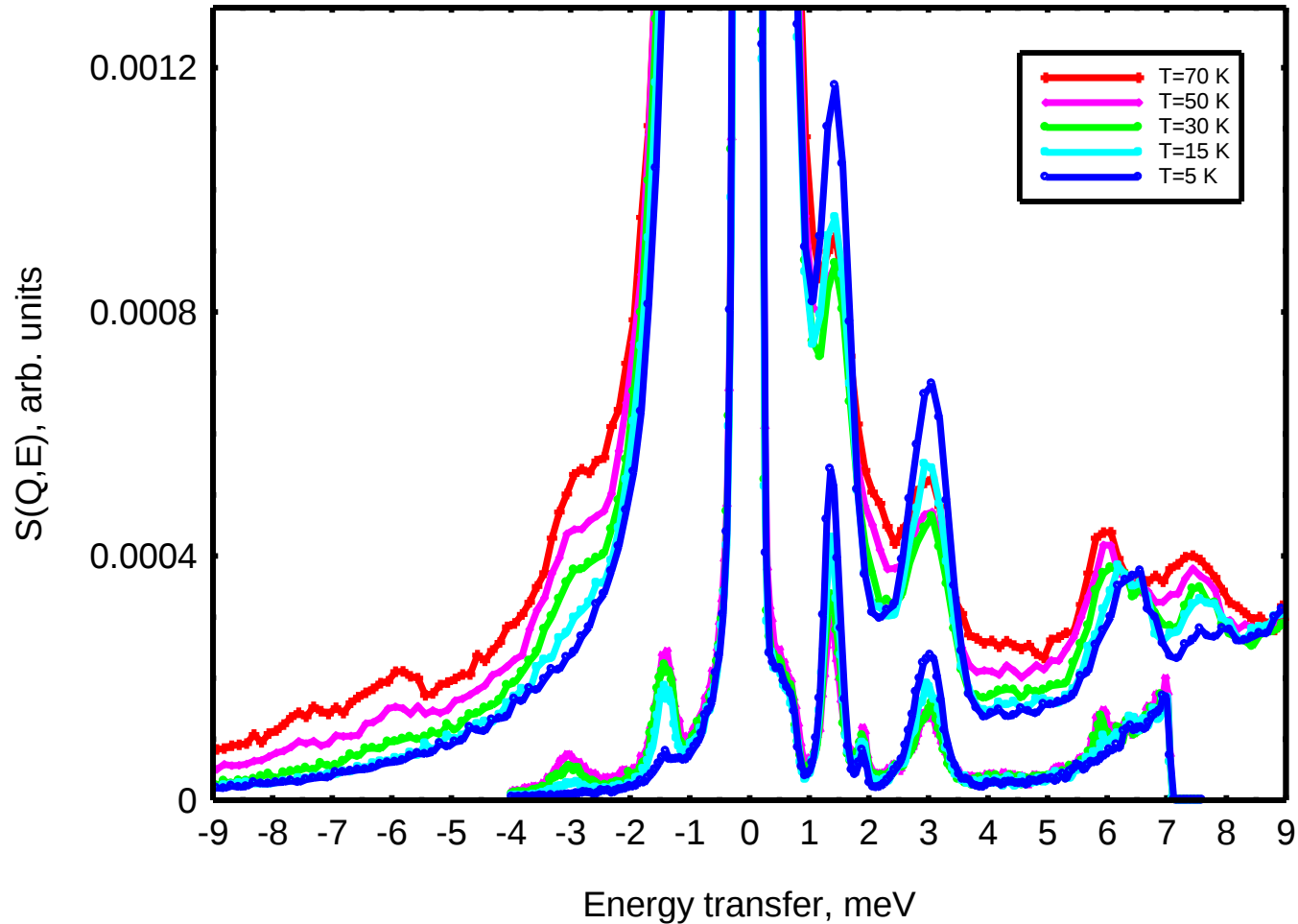


Cordierite

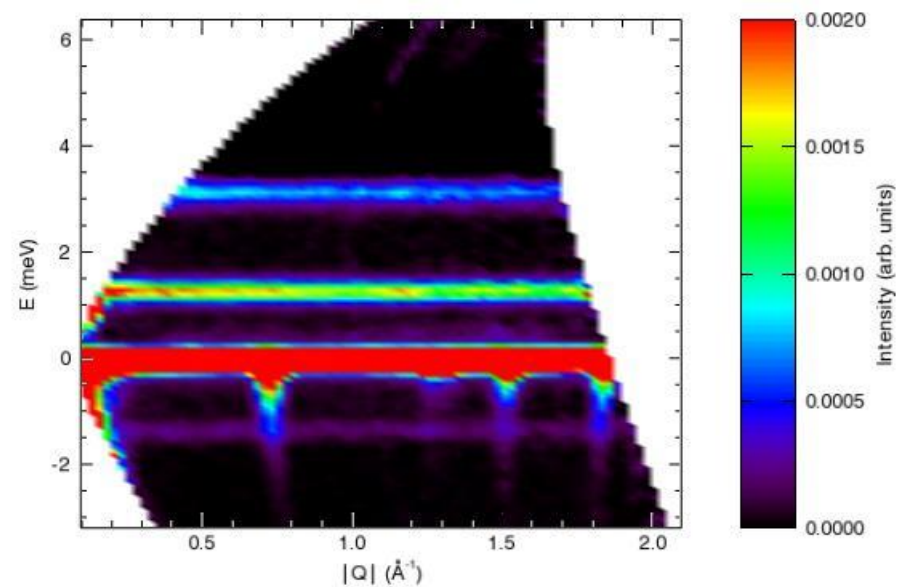
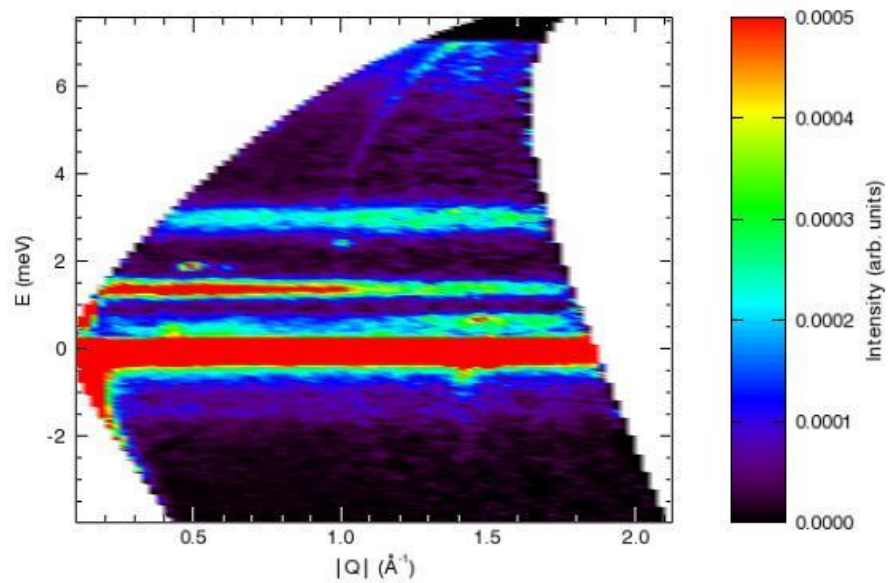


Difference between the $S(Q, E)$ spectra for $Q_{\perp c}$ and $Q_{||c}$ orientations. The solid lines show the neutron recoil spectra for free particles of mass $m=1$ a.u., $E=\hbar^2 Q^2/2m$.

$E_i=8$ and 25 meV



The peaks at 1.4 and 3.0 meV are due to transitions between the levels of the ground state of iron atoms impurity, which are split due to crystal field effect. The intensity of these peaks at energy loss side decrease with temperature increase.



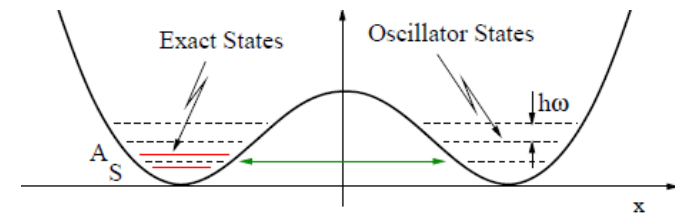
Cordierite and dry cordierite, $E_i=8$ meV, $T=5$ K

The water librational vibrations in beryl have a broad peak at relatively low energy, $E_{lib} \approx 22$ meV (the INS result), and for simplified model of 1D harmonic rotational oscillator, the kinetic energy of water proton in the ground state can be estimated as $E_k = E_{lib}/4 = 5.5$ meV. The De Broglie wavelength, $\lambda = \frac{h}{Mv} = \frac{h}{\sqrt{2ME_k}} \approx 3.8$ Å for this quasi-1D proton (with only one rotational degree of freedom) is much larger than the distances between the proton's possible positions in the cage, which are about 0.6, 1.04 and 1.2 Å, therefore proton in this state might experience quantum tunnel between them, resulting in multiple splitting of its ground state. The splitting of the water proton ground state due to its tunneling can be roughly estimated by using approximate formula [1] for the particle in harmonic double-well potential with minima separated by $l = 0.6$ Å

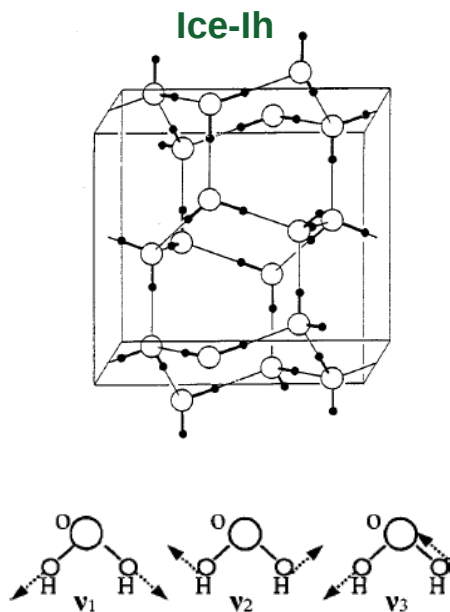
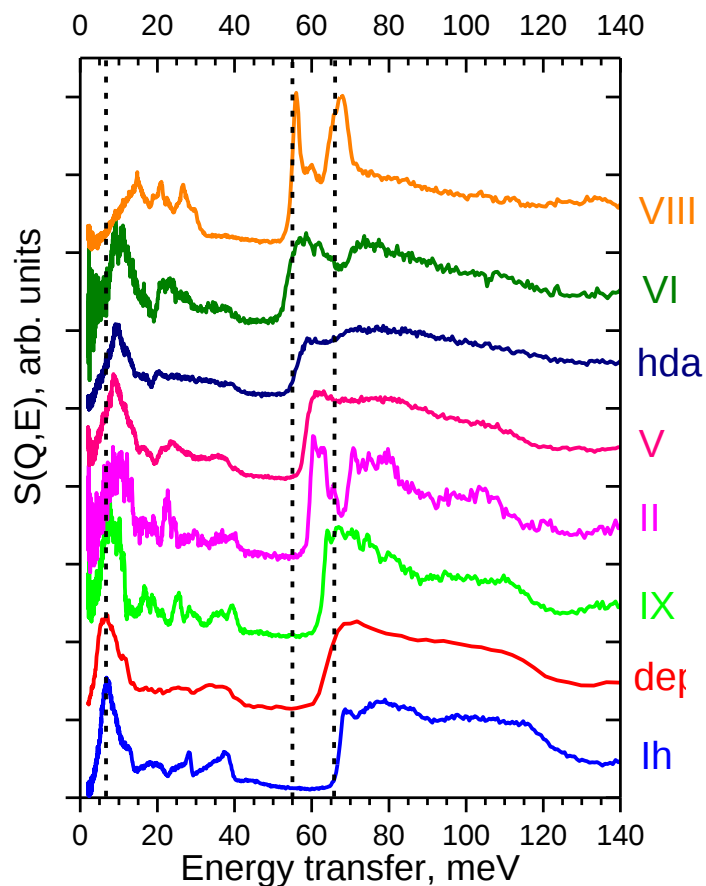
$$\Delta E_{tun} \approx \frac{E_{lib}}{2} \exp\left(-\frac{ME_{lib}l^2}{\hbar^2}\right)$$

which gives a value (~ 7 meV) comparable to the energies of the observed tunneling peaks.

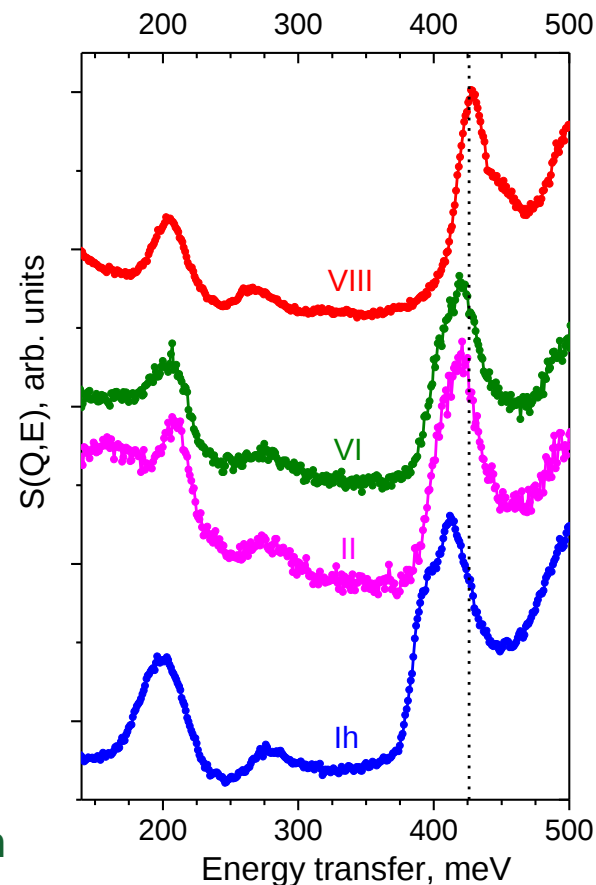
[1] S.L. Drechsler *et al.* J. Phys. F: Met. Phys. 14 (1984) L243



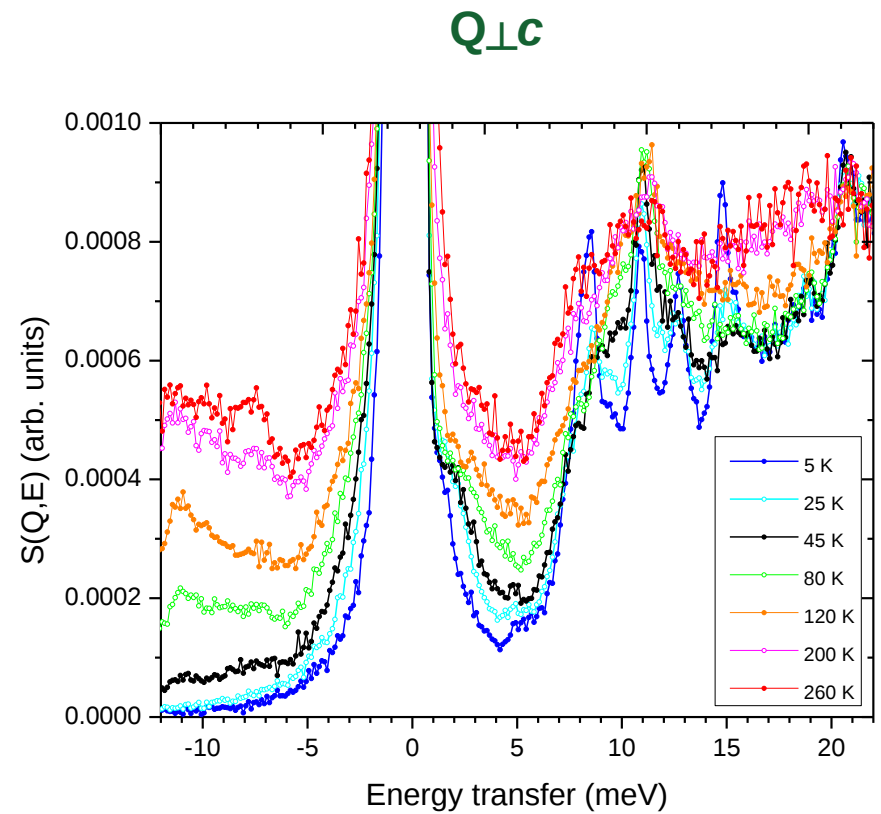
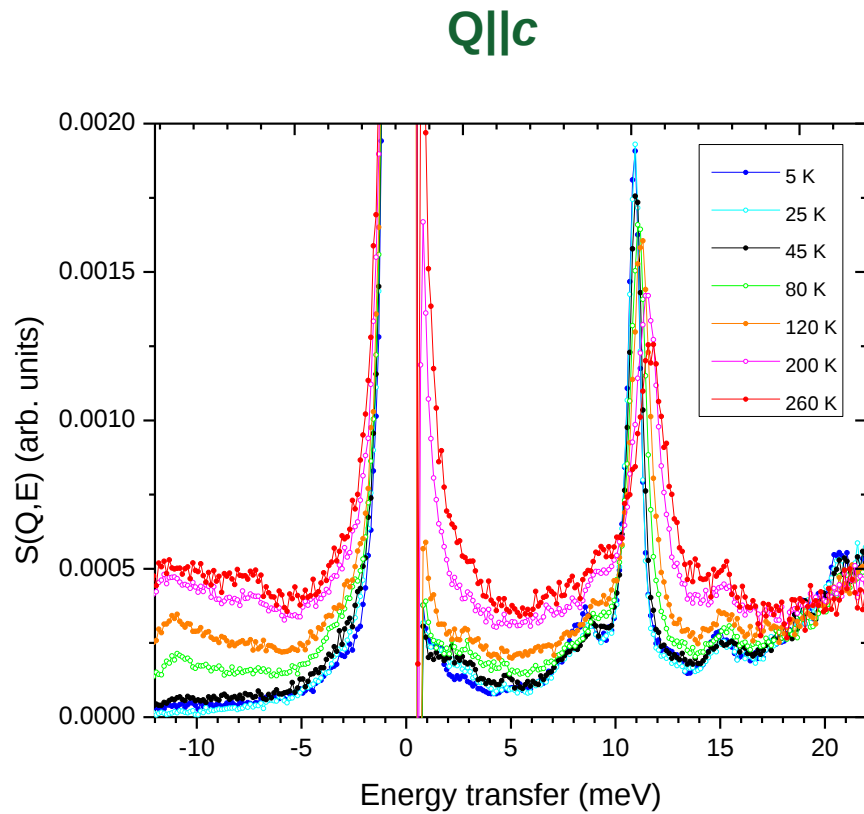
INS spectra of different ice phases

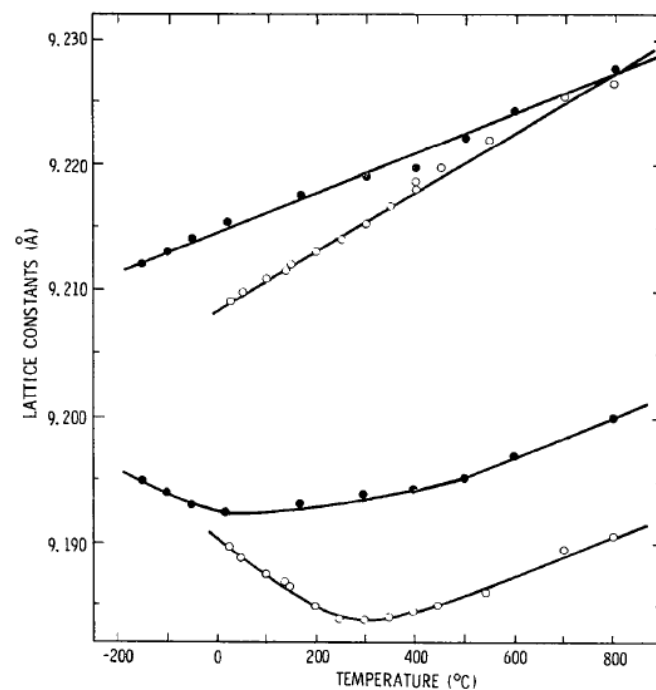
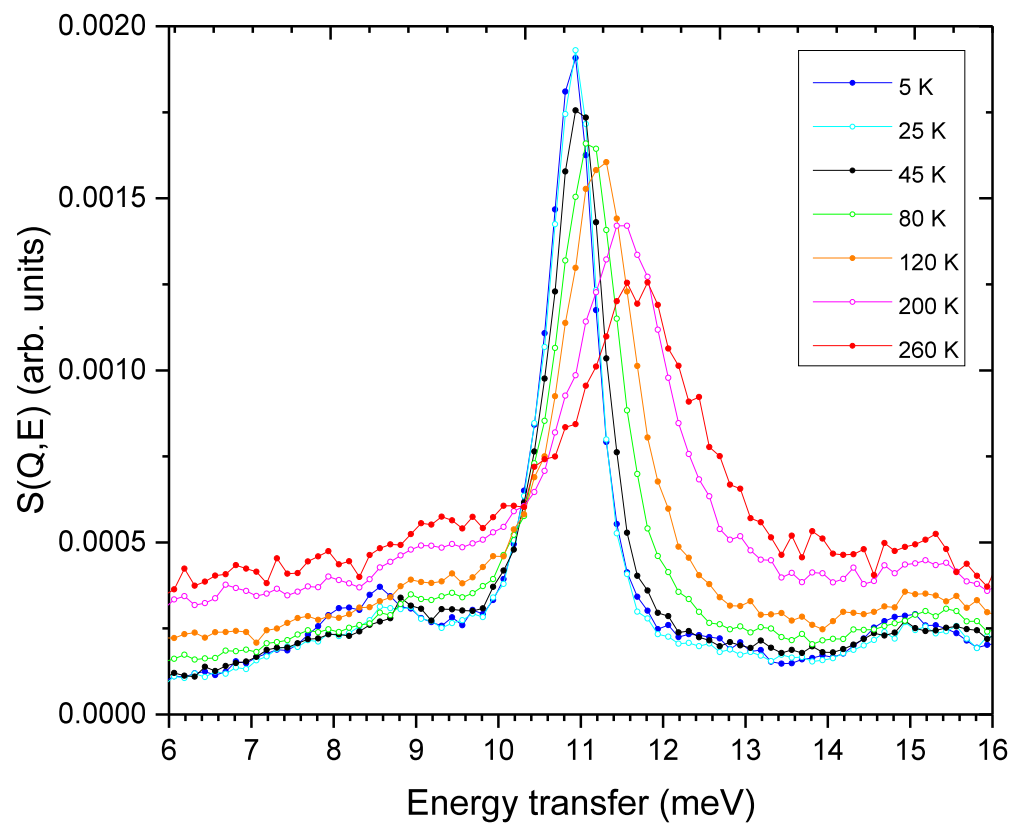


**Schematic illustration
of the bending and
stretching modes in
water molecule.**



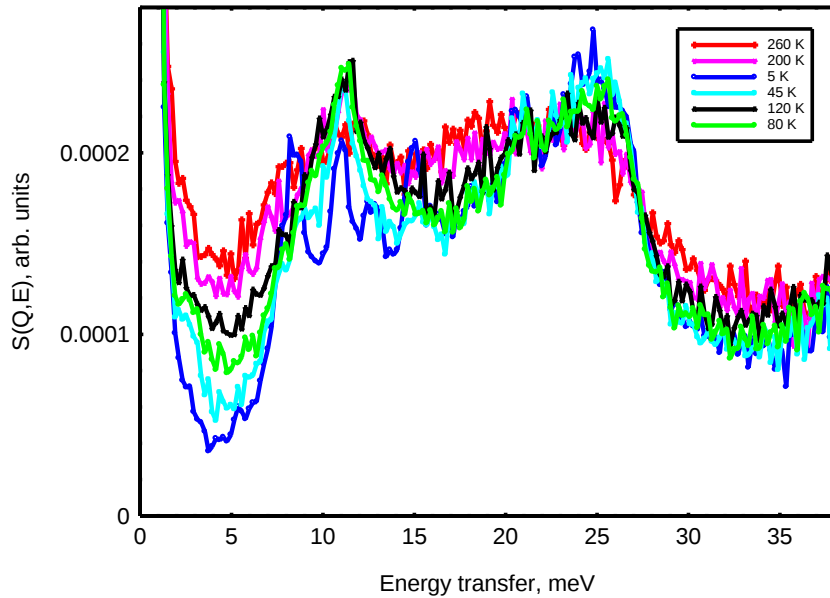
INS spectra of water in beryl, $E_i=25$ meV, measured at different temperatures



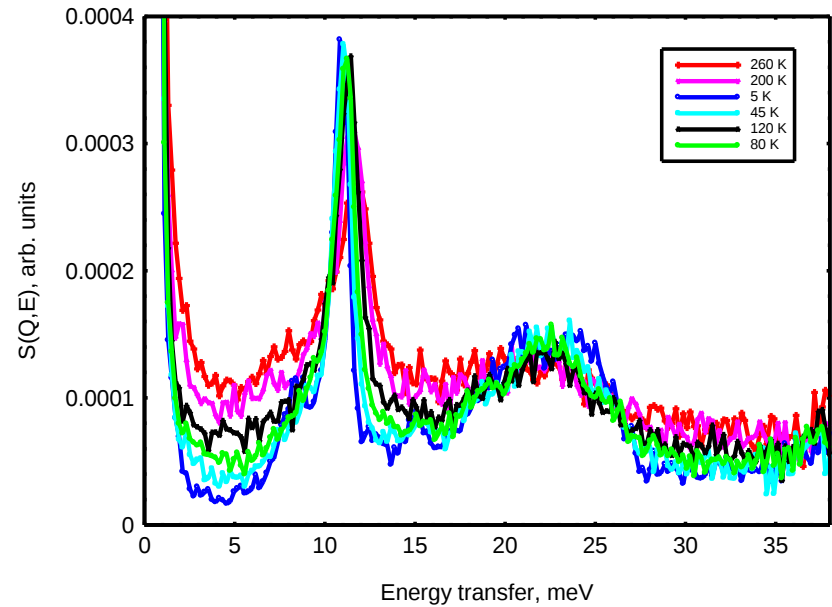


Lattice constants as a function of temperature for beryl and emerald. The values for beryl are shown as open circles, for emerald as solid circles. Note that both crystals exhibit a negative thermal expansion along the c-axis. B. Morosin, *Acta Cryst. B*28 (1972) 1899.

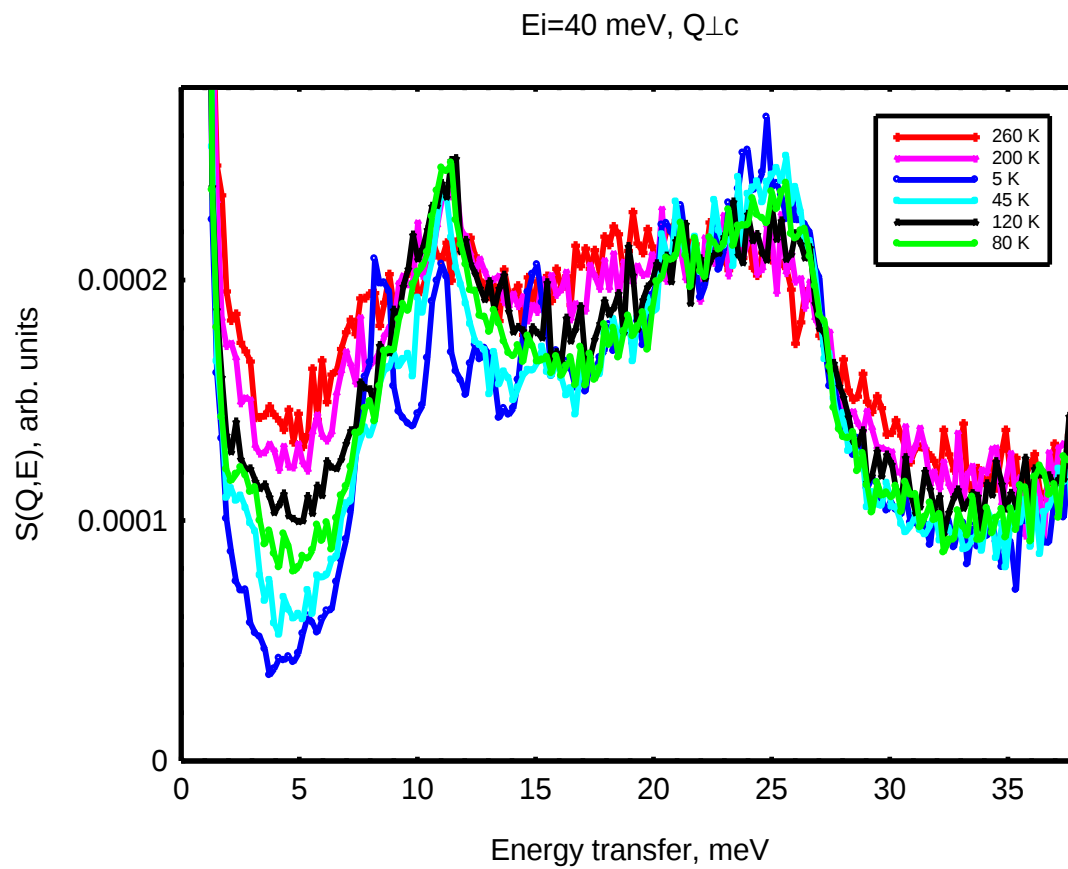
$E_i=40$ meV, $Q \perp c$

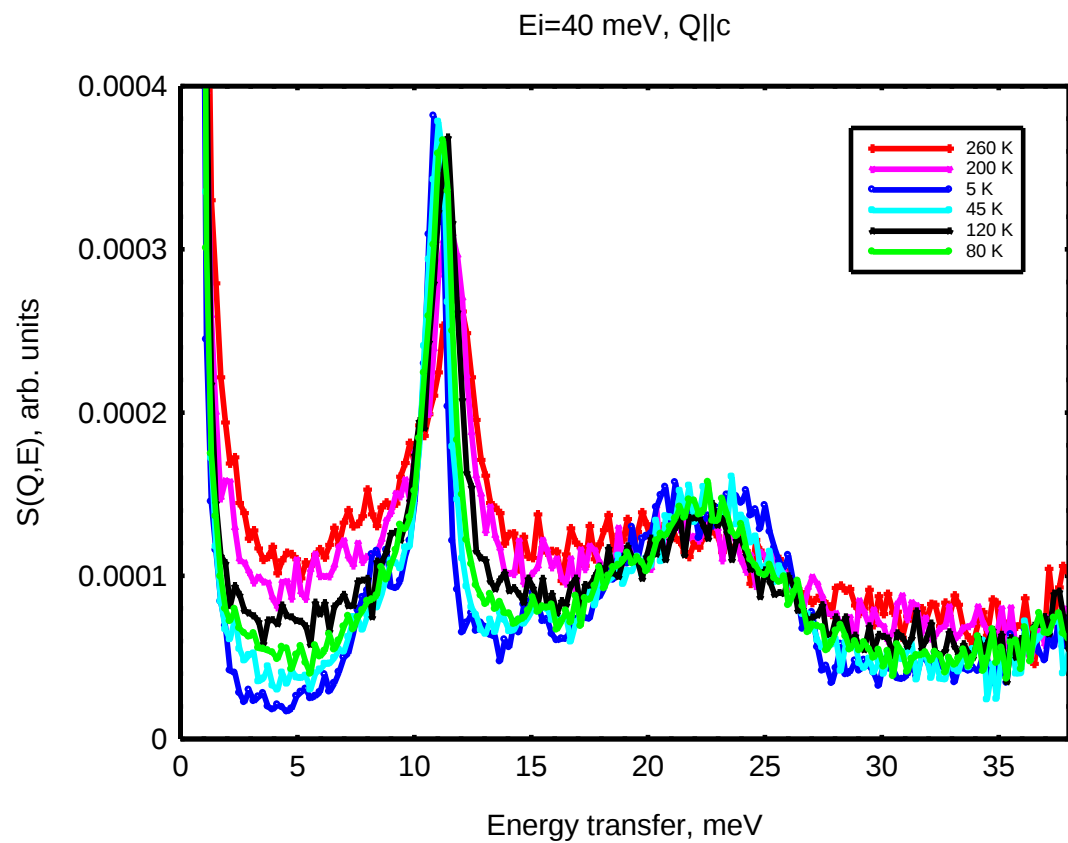


$E_i=40$ meV, $Q \parallel c$

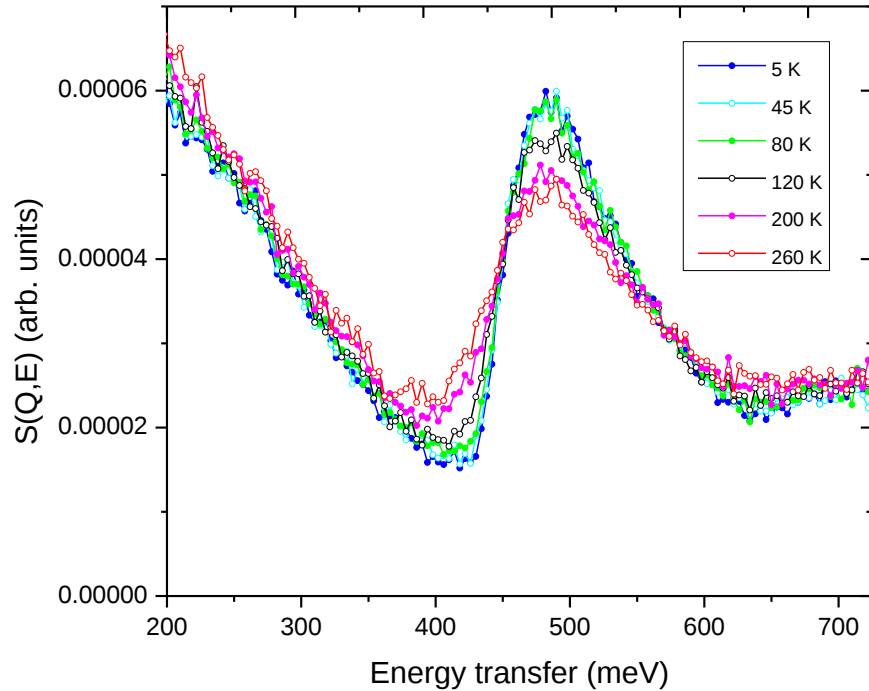


Temperature dependence of INS spectra for water in beryl measured with $E_i=40$ meV for $Q \parallel c$ -axis (left) and $Q \perp c$ -axis (right) orientations.

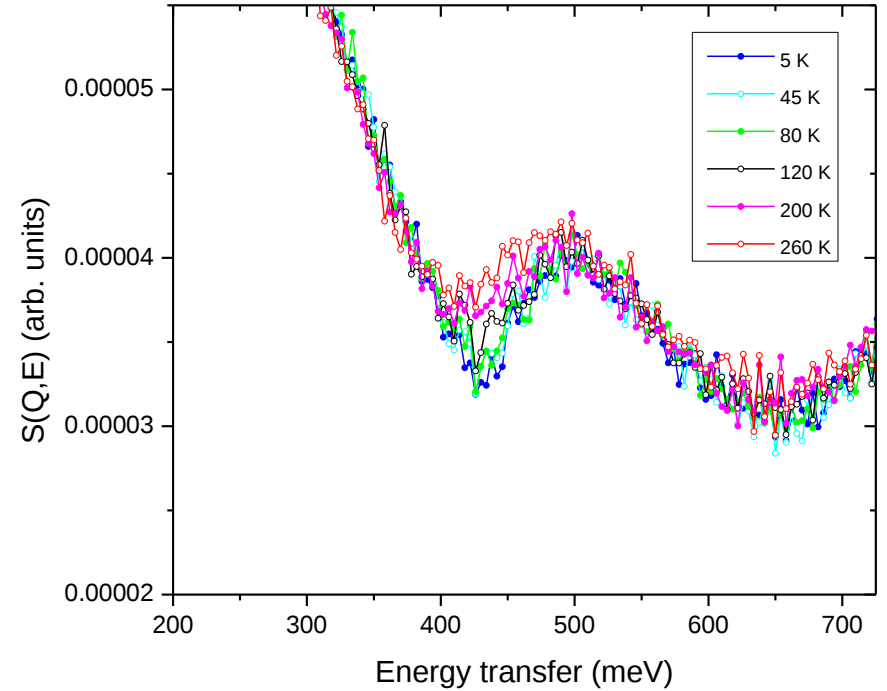




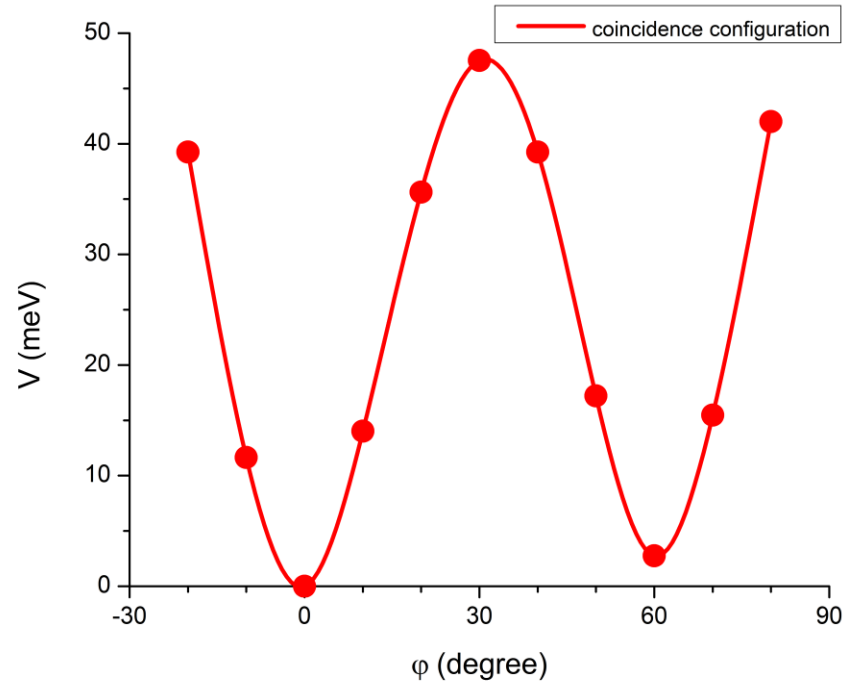
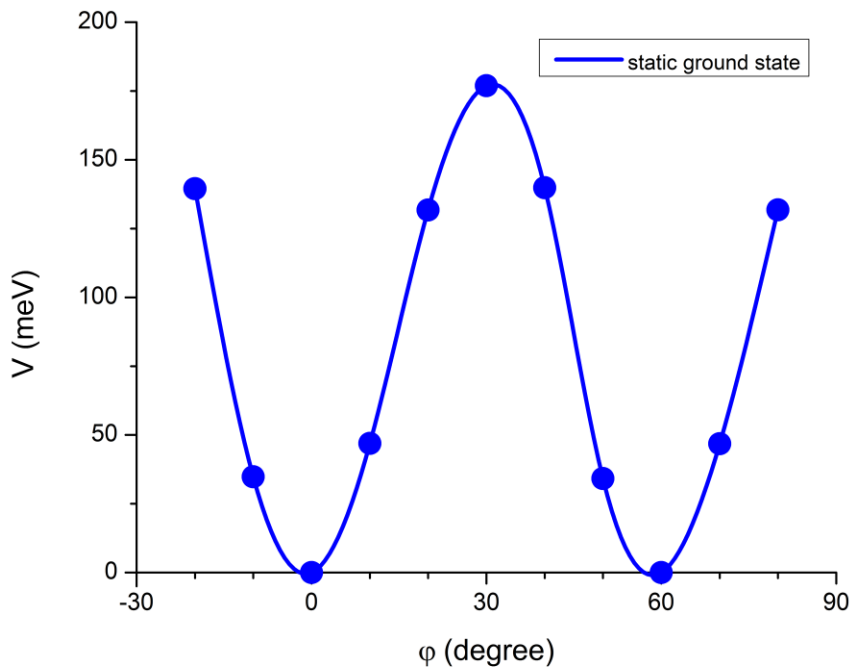
$Q \parallel c$



$Q \perp c$



INS spectra of beryl showing softening of the water stretching O-H modes with temperature increase; $E_i=800$ meV.



(a) The *ab initio* DFT calculated potential for rotation of water molecules about the *c*-axis starting from a static ground state equilibrium structure. The beryl lattice was allowed to relax, and quantum fluctuations were neglected in the DFT calculations. (b) The computed potential for rotation of water molecules about the *c*-axis using the adiabatic coincidence configuration (used in PIMD calculations).

$$H = -\frac{\hbar^2}{2I} \frac{\partial^2}{\partial \varphi^2} + V(\varphi) \quad V(\varphi) = \frac{V_6}{2} (1 - \cos 6\varphi)$$

$$H_{nm} = \frac{\hbar^2 n^2}{2I} \delta_{n,m} + \frac{V_6}{4} \{2\delta_{n,m} - \delta_{n-m+6,0} - \delta_{n-m-6,0}\}, \quad n, m = 0, \pm 1, \pm 2, \dots$$

m	E_m (meV) $V_6=48$ meV	ΔE (meV) $V_6=48$ meV	E_m (meV) $V_6=56$ meV	ΔE (meV) $V_6=56$ meV	ΔE (meV) average	Transitions ($m=0$) $\rightarrow$ (m')
0	21.42		24.51			
1, 5	24.16	2.74	27.16	2.65	2.7	0 $\rightarrow$ 1, 0 $\rightarrow$ 5
2, 4	31.81	10.39	34.40	9.89	10.1	0 $\rightarrow$ 2, 0 $\rightarrow$ 4
3	38.73	17.31	40.51	16.0	16.7	0 $\rightarrow$ 3

The rotational energies E_m for water protons in beryl calculated by using six-fold potential with $V_6=48$ and 56 meV. The energies of the intraband ground state transitions ΔE , related to the tunneling peaks in INS spectra, and their assignments are shown.